PHYSICS 526 NOTES: FLUID DYNAMICS

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Our goal in this course is to explore the basic physics of fluid systems, both with and without the effects of magnetic fields. Fluid physics by itself – hydrodynamics – applies to any neutral fluid (think about water, or molasses, or the earth's atmosphere), whether or not there are magnetic fields around. "Hydro" or "HD" has broad applications – smooth flows, turbulent flows, shocks, sound waves, instabilities. If the fluid is ionized (think of liquid sodium, the higher reaches of the earth's atmosphere, or just about any astrophysical system you can bring to mind), it can (and almost certainly will) carry a current. The interaction of the current with a magnetic field (external or self-generated) modifies all of the phenomena above and adds some new ones. This is described by magnetohydrodynamics (MHD).

Traditionally, HD and MHD are treated separately, but they have a lot in common, and I have learned a lot by comparing and contrasting HD and MHD phenomena. I'm hoping to share that with you as we go through this course.

A word of caution: you should especially note *units and dimensions*. These notes are in cgs. That makes very little difference for "rocks" (analyses that involve mass, length, time); but it makes a big difference for electrodynamics and MHD. The $\mathbf{E}$ and $\mathbf{B}$ fields, as well as the fundamental charge, have different dimensions in cgs than in SI; and the coupling constants in Maxwell's equations are different. Both systems appear in the literature; while there is some trend towards SI, many important references are in cgs. I am going to follow my experience and preference and use cgs; I'll also give the SI versions of most critical equations.

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