11 Accretion II: compact objects

Now, let's get back to accretion onto compact objects – dense stellar remnants and black holes.

11.1 Basic ideas

Before we talk about specific objects, let's start with an overview of important physical ideas.

11.1.1 Energetics ("Accretion Power")

Recall what we discussed briefly in chapter 6. The basic idea is that accreting matter liberates its gravitational energy, at a rate $\dot{E}_g \sim GM\dot{M}/r$, for some mass accretion rate $\dot{M}$ and some "conversion radius" r. Physically, this energy must be converted to internal energy of the accreting stuff (*via* friction, for instance) and the hot matter then radiates with some (as yet unspecified) efficiency ε:

$$L = \varepsilon \frac{GM\dot{M}}{r} \tag{11.1}$$

This applies to general accretion flows, be they onto YSO's or neutron stars.

Another approach is sometimes used for accretion onto black holes. Let's think about the conversion radius r. The smallest it's likely to be is some factor times the "gravitational radius" around a black hole, $r_g = GM/c^2$.[1] If we scale r in (11.1) to r_g, we have

$$L = \varepsilon \frac{GM\dot{M}}{r_g}\frac{r_g}{r} = \varepsilon \frac{r_g}{r}\dot{M}c^2 \tag{11.2}$$

Thus, for conversion close to r_g, ε measures the output luminosity as a fraction of the infalling rest mass energy; this scaling is often used in black hole accretion models.

What are typical numbers? A typical galactic X-ray binary might have $L \sim 10^{37}$ erg/s; this requires $\varepsilon\dot{M} \sim 3.5 \times 10^{-7} M_\odot$/yr, for conversion close to r_g. Or, a bright quasar might have $L \sim 10^{46}$erg/s; this requires $\varepsilon\dot{M} \sim 0.35 M_\odot$/yr.

11.1.2 Eddington luminosity

This is an important reference point: at what luminosity can the radiation pressure from a central source of luminosity L can offset gravity from a central mass M, and stop the flow? For most accretion problems, the infalling matter is fully ionized hydrogen. That means we must consider the gravitational force on a proton, and note that the radiation pressure is communicated *via* Thomson (electron-photon) scattering (which has a cross section $\sigma_T = 6.65 \times 10^{-25}\mathrm{cm}^2$).[2] With this we can derive the Eddington luminosity:

$$\frac{L_{edd}}{4\pi r^2}\frac{\sigma_T}{c} = \frac{GMm_p}{r^2} \;\Rightarrow\; L_{edd} = \frac{4\pi GMm_p c}{\sigma_T} \tag{11.3}$$

and numerically, this is $L_{edd} \simeq 1.3 \times 10^{38}(M/M_\odot)$ erg/s.

A related quantity is the *Eddington mass flux*. This is the mass flow that produces L_{edd} in a given system. Defining

$$L_{edd} = \varepsilon \frac{GM\dot{M}_{edd}}{r}$$

gives us

$$\dot{M}_{edd} = \frac{4\pi}{\varepsilon}\frac{r}{r_g}\frac{GMm_p}{c\sigma_T} \tag{11.4}$$

11.1.3 Thermal state

This is critical to interpreting observations of accretion systems. While the full story here can be quite complex,[3] one useful simple estimate is possible. If the infalling matter *and radiation it generates* are in something close to thermal equilibrium, we know from basic thermodynamics that it radiates as a black body. That means the luminosity coming from its surface has an intensity (energy per time per surface area) $\sigma_{SB}T^4$, where $\sigma_{SB} = 5.67 \times 10^{-5}$ (cgs) is the Stefan-Boltzmann constant. We also know that the typical photon energy $h\nu \sim kT$. Equating the luminosity in (11.1) to that lost by black body radiation determines the lowest temperature the gas will reach. You will find that $T \sim 10^7$ K for accretion onto a solar-sized black hole (and thus we have galactic X-ray binaries); and $T \sim 10^4 - 10^5$ K (and thus we have UV and optical sources) for accretion onto a massive $M \sim 10^9\ M_\odot$ black hole (as in a galactic nucleus).

We must remember, however, that the situation in an accretion flow is rarely that simple. One complication

[1] Think about the event horizon of a black hole, or the smallest stable circular orbit – both of these are $\propto r_g$. We'll return to this later in this chapter.

[2] Other physical states for the accreting matter call for other values of σ and m.

[3] We would need to know how rapidly the plasma is heated by friction, turbulence, magnetic dissipation, etc; and how effectively it can lose energy by radiation – which depends on all sorts of details about the plasma density, temperature, etc..

is that the radiation emitted by the infalling matter may well not be in thermal equilibrium with the matter; that will be the case if the flow is transparent to the radiation, meaning that the mean free path for a photon to be absorbed or scattered is large compared to the size of the system. Just how the radiation comes out will be a major topic for next term. For now, I'll just note that the inflowing plasma will be *hotter*, and transparent (parts of) accretion flows can reach temperatures $\sim 10^8 - 10^{10}$ K. Another complication is that some of the accretion energy does not simply to go heating, but (somehow) goes to accelerating a few particles to relativistic energies. We know this because we know that many accretion sources are "nonthermal", with significant emission by these relativistic particles due to their gyromotion in the local magnetic field.

11.1.4 The transtion to disk accretion

We've worked so far with spherical accretion – because it's simple, and because it's probably a useful limiting case (for accretion from large distances in a quasi-symmetric system). But think about conservation of angular momentum. If the quasi-spherical inflow has any net angular momentum at all, its angular velocity will increase as the matter flows inward; eventually we can no longer assume spherical inflow, and rotation will dominate *perpendicular to the rotation axis*. However, parallel to the rotation axis the collapse can continue – and in most systems will be helped along by radiative cooling (as the collapsing matter loses its internal energy and pressure support against gravity). Thus we expect an inflow that is initially spherical – say at large distances from its core – to change to disk-like accretion closer to the core.

From there, the matter must lose angular momentum in order to keep accreting. This is thought to come from viscosity in quasi-steady flows (as we'll see in chapter 12), or possibly from instabilities in unsteady flows (*i.e.*, the flow might fragment into blobs, some of which move inward).

11.1.5 Size Matters

The size of the accretion region is critical to both the energetics and the thermal state of the accreting matter. That means it's critical to both the luminosity and the spectrum of the accretion flow. For quick reference, the interesting sizes are:

• The *YSO radius* is probably comparable to the radius the YSO will have when it reaches the main sequence. If the YSO is strongly magnetized, however, magnetic pressure may stop the accretion flow at a significantly larger radius.

• The *neutron star radius* is about 10 km. Most of the action for accretion onto a neutron star probably occurs close to this radius.

• *Gravitational radius* of a star-sized black hole is $r_g \simeq 1.5(M/M_\odot)$ km. Most of the action for accretion onto a black hole probably occurs from a few to a few tens of r_g.

• *Gravitational radius* of a galaxy-sized black hole is $r_g \simeq 1.5 \times 10^{14}(M/10^9 M_\odot)$ cm $\simeq$ 10 AU for a $10^9 M_\odot$ black hole — the radius of Saturn's orbit. So most of the action for a massive black hole in a galactic nucleus occurs well below 1 pc.

11.1.6 Jets and outflows

Finally, we should note that just about every accretion **inflow** involves an **outflow**. This comes from the data: **jets** (and sometimes less collimated outflows as well) are found in connection with just about every accretion disk, everywhere. This has long been known to be true for massive black holes in AGN. We now know that many star-sized binary accretion systems within the galaxy drive out well-collimated jets. We also now know (as in chapter 10) that jets are commonly found associated with YSO's, presumably in their disk-accretion phase.

With these general ideas in mind, let's look at some of the objects and astrophysical settings for "classical accretion".

11.2 The Setting: Compact Stellar Remnants

Our focus in this chapter is accretion onto compact objects – neutron stars and black holes. While this is not a course in stellar structure, one of our applications has been the ISM. We remember that the state of being a "star" ties up that piece of the ISM for quite while. Let's bypass that and jump from YSO's to the very end of the star's life.

11.2.1 From main sequence stars to remnants

We can recall the likely events at the end of a star's main sequence life, after it has exhausated its nuclear fuel and nuclear burning has stopped. Its evolution depends on its mass. One scaling mass is the Chandrasekhar mass, $M_{ch} \simeq 1.4 M_\odot$; this is the maximum mass of a star which can be supported by electron degeneracy pressure. Current thinking as to the end of a star's life is as follows – though note this picture is vastly oversimplified:

• $M \lesssim M_{ch}$: the star quietly settles to a state supported by electron degeneracy pressure, that is a white dwarf.

• $M_{ch} \lesssim M \lesssim 8M_\odot$: the star develops an electron degenerate core, at $M \sim M_{ch}$, which becomes a white dwarf. The rest of the star's mass is ejected, in some form such as a strong stellar wind or a planetary nebula.

• $M \gtrsim 8M_\odot$: the star meets a violent end. When its fuel is exhausted, the core collapses suddenly. This collapse drives the core through the electron degeneracy density, into a more dense and more compact remnant. The outer layers of the star "bounce", and are driven outwards at high speed. This is a Type II supernova.[4] Detailed models of SN currently predict that the remnant is a neutron star if the original star's mass is $\lesssim 30 M_\odot$, and is a black hole for larger original masses.

We should note that these arguments are based in the best current stellar evolution models – but that they are still somewhat uncertain (the number "8" really means "several to 10"). In addition, there is still a discrepancy between the supernova rate in the galaxy and the required pulsar birth rate – so we don't understand everything yet. What we can say with certainty, is that the normal process of stellar evolution ties up some of the total (baryonic) mass of the galaxy into remnants – white dwarfs, neutron stars and black holes – which just sit there, providing gravity but not interacting with the ISM or galactic evolution any further.

11.2.2 The result: (star-sized) compact objects

Our interest here is the connection between these stellar remnants and accretion processes in astrophysics. To that end, I'll group them by how we observe them, and/or their role in high-energy astrophysics.

• **White dwarf stars** are the low-mass end of the compact-object set. We are not going to say much about them...they are not as important in high-energy astrophysics.

• **Isolated neutron stars** can appear as pulsars. These small (radius $\sim$ 10 km), rapidly rotating stars have strong, narrowly beamed "hot spot" sources of coherent radio emission; when the beams rotate into our line of sight we see a "pulse". A few are also pulsed X-ray and γ-ray sources, as the radio emission region seems also to emit beamed X-ray and γ-ray photons. These stars have very high magnetic fields ($\sim 10^{12}$ G), and a dense, corotating charged magnetosphere (probably composed of an electron-positron "pair" plasma). They emit coherent radio radiation – generated by some (as yet uncertain) plasma process in which bunches of charges oscillate together. Finally, it's thought that they drive *relativistic winds*, which carry mass and energy away from the star; these winds may be what feeds the filled supernova remnants (called *plerions*).

• **Neutron stars in binary systems** have many options. Some of these are pulsars, particularly the rapidly rotating millisecond pulsars which are thought to have been spun-up by accretion of matter from their companion. In addition, binary-system neutron stars are often strong X-ray sources – not beamed but more isotropic. The energy source here is very likely quasi-steady accretion, through an accretion disk, from the companion star. In the accretion process, gravitational energy is turned into heat (possibly also relativistic, nonthermal particles), and from there goes to radiation. Jets are also common – but not universal – in such systems (such objects as SS433, or the "microquasars" being found recently). As with protostellar jets, the existence of jets here reinforces the jet-accretion disk connection.

• **Black holes in binary systems** can also be accretion-powered X-ray and γ-ray sources. Just as with neutron star binaries, the radiation in BH binaries comes from the the accretion process; the fact that the "action" is occuring at somewhat smaller radii should lead to some differences in the details. The most important point about these systems is probably that they exist. That is, the parameters of the binary orbit allow us to determine the mass of the compact, accreting companion. There are a handful of systems for which the compact object's mass is comfortably above $3M_\odot$ (the theoretical "Chandrasekhar"-type upper limit on the mass of a neutron star) — and for these systems, we can argue strongly that they contain a star-sized black hole.

[4] You recall the two types of supernovae. Type I are believed to come from accretion of matter onto a white dwarf, which drives a thermonuclear explosion. They are very uniform in their spectra and their light curves, with potential use as standard candles. Type II are much more varied in their properties, and are thought to come from stellar collapse as described above.

• Finally, an historical comment. **Gamma-ray bursters** used to be included in this list. Until a few years ago, people argued as to whether they are galactic or extragalactic (there was no clear measure of their distance). Mose people favored a galactic location; at such a distance, the energy of their bursts suggested some explosive accretion event involving a neutron star. Recently, however, it has been shown conclusively that they are extragalactic (from the high redshift of spectral lines in the associated optical sources; finding such sources was a major step forward). This pushes their energies up by a lot – if they were isotropic emitters, the energy released would approach the regime where models might involve an explosive release of the entire binding energy of a neutron star (as in a NS-NS collision?); larger than the canonical 10^{51} erg known to be released in a supernova. But again, another important observation clarified this: a brand new supernova was found at the exact spot where a GRB had just gone off. How can we reconcile the energetics? It can all work if the GRB is *beamed* – if part of the supernova process is the creation of a short-lived relativistic jet of material. Relativistic effects make the γ-radiation emitted by this jet appear much brighter when viewed close to the jet's direction of motion – that's called *forward beaming*. And yet the game isn't over: there are two types of GRBs (short pulse and long pulse); only one type seems to be consistent with SN explosions; the origin of the other type is still being discussed.

11.3 The Setting: Active Galactic Nuclei

Black holes in galactic nuclei aren't really "stellar", but they are related to star-sized compact objects in the ways they shine (and the ways in which they are modelled). Active galactic nuclei – Seyfert galaxies, radio galaxies, quasars – are believed to be powered by accretion onto supermassive black holes ("SMBH"; $\sim 10^8 - 10^9 M_\odot$). In a very few objects spectral lines or masers can be localized close to the central mass, with clear signs of ordered (disk-like) rotation. The inferred velocities are used to determine the central mass. It's worth pointing out that this isn't directly a detection of a black hole; it's a detection of a small, "massive dark object" (MDO, the term in some of the literature). Whether MDO or BH, models of AGN generally assume an accretion disk flow, with all the associated physics that can occur in star-sized accretion systems. Some AGN – those in radio galaxies – are also, of course, associated with strong, collimated, relativistic jets. In addition, we now have strong (if indirect) evidence that *every* galaxy contains a MDO in its heart – with mass related to the mass of the "bulge" part of the galaxy. But in most galaxies the MDO is not "active"; we detect it only by its gravitational effects.[5] More on this next term.

11.4 Black Holes (a quick visit)

This isn't a course in general relativity, so these notes are not the place for a full exposition of general relativity. I will assume that you have seen the basics, such as in Carroll & Ostlie, and will focus on those aspects of black holes which are relevant to their role in high-energy astrophysics. We will only have a once-over-lightly visit, emphasizing the aspects of black holes that are important for their accretion-related astrophysics.

In terms of accretion physics – or the black hole's impact on its surroundings – we only need to understand a few critical radii. We need to know the size of the *event horizon* – that's the surface inside of which nothing (not a rock, a photon, you or me) can escape. We also need to know a little bit about *stable orbits*, as follows.

11.4.1 Stable orbits

To set the stage, think about circular orbits around a mass M in Newtonian gravity. They're easy: gravity provides the centripetal force. Thus, $GM/r^2 = v^2/r = L^2/r^3$ connects r to v (or to the angular momentum per mass, $L = rv$), uniquely. At any radius r, the orbit is described by $L^2 = GMr$. In addition, you know this orbit is *stable*: think about perturbing a planet in orbit around the sun. It will just oscillate radially around its initial radius – that's an epicycle. In other words, any circular orbit in Newtonian gravity is stable. (This also holds for closed elliptical orbits, the math just gets longer). You also know about open (hyperbolic) orbits: if I start at infinity and throw a rock at the sun, and it has any angular momentum at all, it will pass by the sun and escape back to infinity. It won't be captured unless I drop it directly at the sun (that means it has zero angular momentum).

However this changes in General Relativity. At large radii we can indeed find a stable circular orbit; and as $r \to \infty$, the solution approaches the Newtonian one.

[5] Does this mean it's not accreting matter? If so, why not?

But for small radii, *we can no longer find a stable circular orbit*. If you try to put a rock in orbit close to the black hole, it will either fall in or fly away. You won't be able to find an equilibrium. The *innermost stable radius* is critical for disk accretion – it's the smallest radius for which quasi-stable accretion disks can exist. This innermost radius is also relevant to *capture orbits*: if a rock comes too close to the black hole, it will be captured (it's trajectory will pass within the event horizon), even if it has finite angular momentum. The capture radius is usually comparable to the innermost stable radius although it has to be derived by more complicated methods.

Now, we carry on to describe (not derive!) black hole solutions of Einstein's field equations.

11.4.2 Schwarzchild black holes

Formally, a "black hole" is the name we give to a particular vacuum solution of Einstein's field equations. The fundamental quantity in classical GR is the metric: the distance between two space-time points. The GR field equations are a set of second-order PDE's which describe the metric and its connection to the sources.[6] Solutions to these equations must assume some type of symmetry, and will involve one or more constants of integration.

Two important vacuum solutions have been worked out.[7] The simplest is an isotropic, static solution, which has one constant of integration (we call it M). This is the **Schwarzchild metric**. It is the most familiar: it describes the space around a non-rotating object of mass M.

A Schwarzchild black hole has one critical surface, the *event horizon*, at $r_s = 2r_g = 2GM/c^2$. This is the surface of no escape; no trajectory (particle or photon) that starts within r_s can reach the outside world. It's the surface of infinite time dilation: periodic signals starting at r_s are dilated to infinite period. It's the surface of stationarity: outside of r_s, you can sit still (think of firing your rocket motors "downward", to hold yourself in a fixed position relative to the star), but inside r_s, this is not possible. (Mathematically, it's easy to show that $dt > 0$ – advancing time – requires $ds < 0$ – motion in space – inside of r_s). However, you should note that r_s is not in any way a physical barrier for something moving inwards; except for tidal forces, you don't notice anything dramatic as you cross the event horizon. The unusual effects are related to how you or your signals connect with the outside world.

What about astrophysical applications that do not involve a mass crossing the event horizon? This is where we need to analyze orbits in this geometry. When this is carried out, we find there is a *minimum* radius for which stable circular orbits are possible. It is:

$$r_{ms} = 6r_g = 3r_s \tag{11.5}$$

No stable orbits are possible at smaller radii. This is, thus, another important scale for a Schwarzchild black hole; it is often taken as the inner edge of an accretion disk around such a BH.

11.4.3 Kerr black holes

The next simplest is an axisymmetric, static solution, which has two constants of integration (M and J). This is the **Kerr metric**. It describes the space around an object of mass M and angular momentum J. Notation: it's common to work with the parameter $a = J/Mc$, the normalized angular momentum per mass. Well-behaved solutions exist only for $a < M$. Because of this mathematical limit,[8] the astrophysical speculation is that more rapidly rotating systems can never become black holes.

A Kerr BH has two important scaling radii. It has an *event horizon*,

$$r_o = \frac{r_s}{2} + \left[\left(\frac{r_s}{2}\right)^2 - a^2\right]^{1/2} \tag{11.6}$$

which is the surface of no escape, just as in the Schwarzchild case. Note this is a spherical surface,

[6] Want a familiar example? Think of Maxwell's equations: they are PDE's with the field terms $(\mathbf{E}, \mathbf{B})$ "on the left", and the source terms $(\rho, \mathbf{j})$ "on the right". The GR field equations are analogous – the terms on the left involve the metric; the terms on the right involve the sources, namely, the distribution of mass-energy. A vacuum solution is analogous to a point charge in empty space – no external sources.

[7] A third solution is the **Reissner-Nordstrom metric**; it is not a vacuum solution, but rather allows a radial electric field to exist throughout space. This solution adds a third constant, Q, corresponding to the object's charge. There is little evidence that any astrophysical body carries significant charge, and good arguments against that being the case (think: if you charged up a star, how long would it stay charged, given all those free charges around in the ISM?) Thus, the R-N solution is rarely invoked; the first two are the common ones.

[8] which is in units with $G = c = 1$, which relativists love; in more normal units, the limit is $J/Mc < GM/c^2$.

also as in the Schwarzchild case. The *surface of stationarity* is distinct in the Kerr metric:

$$r_+ = \frac{r_s}{2} + \left[\left(\frac{r_s}{2}\right)^2 - a^2\cos^2\theta\right]^{1/2} \tag{11.7}$$

You can escape from within r_+, but you can't sit still. The direction of increasing time, inside r_+, is also the direction of increasing ϕ (the angular coordinate). Note that this surface is not spherical; it bulges out at the equator.

An interesting related effect is *frame dragging*. A particle with no angular momentum at infinity will still want to move in the direction of increasing ϕ. The rotation speed of such a particle, seen at r as measured by a distant observer, is in general

$$\frac{d\phi}{dt} = \frac{r r_s a}{(r^2 + a^2) - a^2\Delta} \tag{11.8}$$

where $\Delta = r^2 - 2Mr + a^2$. A more useful form of this can be found by going to the equatorial plane ($\theta = \pi/2$), and taking $r \gg a, M$. Putting all back in physical units, and converting to an angular speed, we have

$$\omega_{LT} \simeq \frac{2GJ}{r^3c^2} \tag{11.9}$$

Orbital mechanics in the Kerr metric are complex. For circular orbits, one again finds that there is a minimum stable radius. Its limits tell most of the story. When $a \to 0$, $r_{ms} \to 6r_g$ (which is the Schwarzchild limit); when $a \to M$ (the maximum possible value), $r_{ms} \to r_g$. These limits are for prograde orbits; retrograde orbits can't get so close in.

Key points

• Accretion energetics;

• Eddington luminosity;

• Thermal state, black body radiation;

• Compact objects, star sized: what they are, how they relate to accretion flows;

• SMBH: where we find them, how they relate to accretion flows;

• MDOs: why can't we assume they're all SMBH?