

8 MHD: more applications

In this chapter and the next, we'll work through some important fluid-type applications in which MHD effects are important. Our focus here will be MHD in the galactic setting. We'll revisit Alfvén waves, and look at how they interact with galactic cosmic rays. We'll also look at magnetic effects on buoyant instabilities, and again apply it to the galactic setting.

8.1 MHD waves, again

First, let's revisit Alfvén waves, which we've seen once already. We again want a more formal derivation: how do the particle motions and magnetic tension (the restoring force here) connect to each other?

8.1.1 Alfvén waves: gory details

You remember these waves, from chapter 5. They are transverse waves, which are not compressive, and which propagate (in the simplest case) along the magnetic field. Thus, they can be thought of as propagating wiggles in the field lines, as in Figure 8.1.

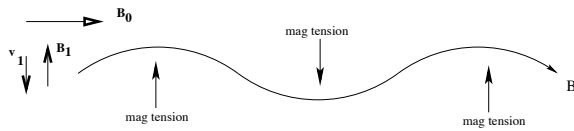


Figure 8.1 Schematic of (shear) Alfvén waves, propagating along a background field \mathbf{B}_0 . The perturbed \mathbf{B}_1 and \mathbf{v}_1 terms are perpendicular to \mathbf{B}_0 . Following Cravens Figure 4.16.

In chapter 5 we used simple, restoring-force arguments to guess their dispersion relation as $\omega = kv_A$, with $v_A = B/\sqrt{4\pi\rho}$. Now let's derive that, more formally, to understand how the waves work. To do that, we'll use the same mathematical techniques as in the last chapter. Consider a uniform fluid at rest: zero velocity, uniform B and mass density ρ . Consider the behavior of a small perturbation – $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1$. Pick \mathbf{B}_0 along \hat{z} , and the wave field $\mathbf{B}_1 = B_1\hat{y}$ along \hat{y} . Because variable \mathbf{B} leads to \mathbf{E} (from Maxwell, right?), we also have the wave electric field $\mathbf{E}_1 \parallel \hat{x}$.

• **Starting points & linearization.** We need to follow both \mathbf{E}_1 and \mathbf{B}_1 , so we need both Maxwell equations:

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}; \quad \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad (8.1)$$

We linearize as usual, picking a perturbation \propto

$e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. Maxwell then becomes

$$i\mathbf{k} \times \mathbf{E}_1 = \frac{i}{c} \omega \mathbf{B}_1; \quad i\mathbf{k} \times \mathbf{B}_1 = \frac{4\pi}{c} \mathbf{j}_1 - \frac{i}{c} \omega \mathbf{E}_1 \quad (8.2)$$

The current density $\mathbf{j} = ne(\mathbf{v}_i - \mathbf{v}_e)$, so we need the equation of motion, for charge q , as usual:

$$m \frac{d\mathbf{v}}{dt} = q\mathbf{E} + \frac{q}{c} \mathbf{v} \times \mathbf{B}; \quad \Rightarrow \quad (8.3)$$

$$-i m \omega \mathbf{v} = q\mathbf{E}_1 + \frac{iq}{c} \mathbf{v} \times \mathbf{B}_0$$

In the last, we've linearized *and* implicitly assumed \mathbf{v} is “small” (so we've dropped $\mathbf{v} \times \mathbf{B}_1$).

• **Particle motion.** We need to follow both electrons and ions. For ions, we keep all the terms in (8.3), and write out both components:

$$-i\omega m_i v_x = eE_1 + e \frac{v_y}{c} B_0; \quad (8.4)$$

$$-i\omega m_i v_y = -e \frac{v_x}{c} B_0$$

These solve to give the ion motion:

$$v_{ix} = \frac{ieE_1}{m_i\omega} \left(1 - \frac{\Omega_i^2}{\omega^2}\right)^{-1}; \quad (8.5)$$

$$v_{iy} = \frac{eE_1}{m_i\omega} \frac{\Omega_i}{\omega} \left(1 - \frac{\Omega_i^2}{\omega^2}\right)^{-1}$$

For the electrons, we can simplify by assuming $\Omega_e \gg \omega$; this gives us

$$v_{ex} = 0; \quad v_{ey} = -\frac{E_1}{B_0} c \quad (8.6)$$

• **Cut to the chase.** These results are all the necessary bits. We put (8.6) and (8.5) into the definition for \mathbf{j} , then put this into (8.2) and combine the two equations there into one (by eliminating \mathbf{B}_1 , say). After a page or so of algebra, we get the dispersion relation for these Alfvén waves:

$$\omega^2 \left(1 + \frac{4\pi n m_i c^2}{B^2}\right) = c^2 k^2 \quad (8.7)$$

With some manipulation, this can be written as

$$\omega^2 = \frac{v_A^2 k^2}{(1 + v_A^2/c^2)} \quad (8.8)$$

where $v_A^2 = B^2/4\pi\rho$, as we saw before (the mass density is $\rho = nm_i$, because the electron mass is so small). Thus, when $v_A \ll c$ (as is usually the case), we have $\omega \simeq kv_A$; or when $v_A \gg c$ (the low-density, high-field limit), we get $\omega \rightarrow ck$.

8.1.2 Magnetosonic waves

Another type of wave is also of interest. Think of a compressive wave, something like a sound wave, but propagating *across* the magnetic field. The magnetic pressure will modify the restoring force; we might expect the wave speed to be a mixture of compressive effects (through c_s) and magnetic effects (through v_A). These are called *magnetosonic waves*.

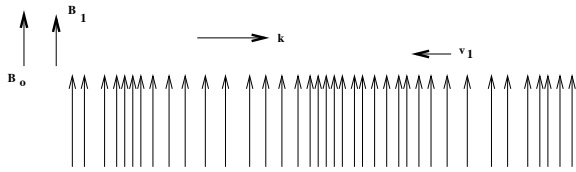


Figure 8.2 Schematic of magnetosonic waves, illustrating a compressive wave propagating at right angles to the background B_0 . Following Cravens figure 4.17.

The full analysis of the wave is messy, and not worth doing here; but *for the simple case of propagation across \mathbf{B}* one finds a wave speed which is what we might well guessed anyway:

$$v_{MS}^2 = c_s^2 + v_A^2 \quad (8.9)$$

Because these waves are compressive, they tend to damp out more easily than Alfvén waves (think of frictional effects in the dense regions). Thus they may be of less interest in general astrophysical situations, than Alfvén waves (which turn out to be only very weakly damped).

8.2 The cosmic ray-Alfvén wave connection

Alfvén waves are particularly interesting because they have resonant interactions with relativistic particles, such as galactic cosmic rays (CR).

8.2.1 Cosmic rays: a quick overview of the observations.

We have already noted that many astrophysical plasmas – including the ISM – contain a significant population of highly relativistic particles which are *not* in a thermal distribution. We have direct and indirect evidence of these particles, which I review very briefly here.

Baryons. Here we have direct evidence – these are the cosmic rays. We can directly measure their flux (including details of composition, energy, charge, and isotropy) at earth; and from there work backwards to

model their distribution above the earth’s atmosphere. (Connecting further back, to their composition outside the heliopause, is harder). Significant facts about their distribution include

- Their energy distribution is a power law, $N(E) \propto E^{-s}$, with a break at $E \sim 10^{15}$ eV (the “knee”), and another at $E \sim 10^{19}$ eV (the “ankle”). The exponent $s \sim 2.7$ below the ankle, and higher above. Comparison of the gyroradius to the scale of the galaxy suggests that the highest energy CR, above the ankle, are extragalactic, while the lower energy ones are galactic in origin.

- Their composition is mostly protons, but there is a heavy element component, with approximately solar abundances (so they come from processed material).

- Cosmic rays are very isotropic in arrival direction, probably at all energies.¹ In terms of their origin, this requires one of three things. (i) The CR are very local, from something like the Oort cloud; this seems unlikely. (ii) If the CR are galactic in origin, they must be isotropized in propagating from their sources (which would lie in the plane of the galaxy) to us. (iii) The CR could be cosmological in origin; this idea has its own difficulties which arise in interactions between the CR and photons of the microwave background.

Leptons. Here we have some direct evidence – the lepton component in the cosmic ray spectrum can be distinguished from the baryon component. The cosmic ray lepton distribution falls much more steeply with energy than that of the baryons, which is probably due to the stronger radiative losses the leptons suffer. We’ll return to this topic next term.

8.2.2 Cosmic rays in the galactic setting

We have two big questions: how are cosmic rays accelerated to such high energies, and how do they interact with the ISM? We’ll defer the acceleration question to next term, but can think here about how they interact with the ISM.

¹NOTE here: this has been an ongoing discussion. Early work suggested that this was the case only at low energies; above the ankle, it had been believed that the highest energy CR were anisotropic, with a tendency to come from the Virgo cluster (remember that’s the cluster that hosts one of the nearest AGN, namely M87). Since then this has come under debate. Some newer work however shows that even the highest energy CR show no strong evidence of anisotropy, but other authors still support an anisotropy.

Sources are thought to be two-fold. *Supernova remnants* have long been thought to be the main source of CR. This is because of their importance in the overall ISM energy budget; because we know they eject heavy elements and the isotopic composition of CR is roughly that of processed stellar material; and because we believe that their outer shocks should be good at accelerating individual particles in the ejecta, up to the relativistic energies typical of CR. In addition, it occurs to me that *pulsars* are probably also an important source. We will see that most models predict pulsars should accelerate single charges to very high energies ($\gamma \sim 10^7$ for instance) close to the star's surface. Some of these particles must go to drive the pulsar wind, but others may well escape directly into the ISM to be a secondary CR source. Still another possibility is that the highest energy CR may have an *extragalactic origin*. This is suggested (i) because of the possible anisotropy, and (ii) because the highest energy ones are unlikely to be “trapped” by the ISM. How these very fast particles are accelerated is one of the major unsolved puzzles – one possibility is that they are generated around the massive black holes in nearby active galactic nuclei.

Propagation. Once generated, CR do not just fly freely through space. Because they are charged, they are connected to the ISM by their gyromotion, and by scattering on turbulent Alfvén waves in the ISM (more details on this, below). Thus the CR distribution we observe at earth may well have been seriously changed, relative to their “birth” distribution, by propagation and scattering through the ISM on their way to us.

Losses. The leptons, being of smaller mass, are susceptible to radiative losses (synchrotron radiation in the galactic magnetic field, inverse Compton scattering on whatever radiation is around) as well as Coulomb losses (scattering on the plasma component of the ISM). This also modifies the electron energy distribution, compared to the source, and of course reduces the net energy in the electron component of the CR. The baryons, on the other hand, don't radiate much – but there is an interesting argument on their confinement lifetime in the ISM, as follows.

Lifetimes. To investigate the confinement of CR baryons, we must look at the evidence on radioactive isotopes and on spallation rates (that is nuclear interactions with ISM protons). The upshot is, that the time an average CR has hung around the galaxy (determined

by radioactive decay) is about ten times longer than the time it has spent propagating through the ISM (determined by spallation rates). From this we learn that an average CR spends most of its life in the galaxy, *not* in the disk, but rather in the more extended halo; and that its lifetime to escape from that halo ~ 20 Myr.

8.2.3 Alfvén waves and wave-particle resonance

We have already seen that cosmic rays are “tied to magnetic field lines”, due to their gyromotion. There is another important effect: Alfvén waves have a strong resonant interaction with the cosmic rays. This interaction can (i) scatter and isotropize the particles; (ii) slow down an initially anisotropic particle beam from $v \sim c$ to $v \sim v_A$ (this is called the “Alfvén speed limit” in the trade); and (iii) possibly accelerate the particles.

Particle-Wave Resonance. Charged particles interact resonantly with Alfvén waves. A particle moving along \mathbf{B} at some velocity v sees a Doppler shifted frequency $\omega' = \omega - kv = \omega(1 - v/v_A)$. Now, the particle will interact with the fluctuating E field of the wave; if the particle “stays in phase” with this fluctuating wave, it will undergo a strong interaction. This happens if the Doppler shifted wave frequency is close to the particle's natural frequency, its gyrofrequency. That is, the interaction is strong when

$$\omega - kv = \pm \Omega \quad (8.10)$$

For relativistic particles, with $v \gg v_A$, this condition solves to an approximate equality between the particles gyroradius and the wave's wavelength:

$$r_L(\gamma) = \gamma \frac{mc^2}{eB} \simeq \lambda_{res}(\gamma) \quad (8.11)$$

Numerically, note that particles with $\gamma mc^2 \sim 1$ GeV, in a field $B \sim 1 \mu\text{G}$, have a gyroradius – and thus a resonant Alfvén wavelength – on the order of an AU.

Wave-particle scattering. Assume, now, that we have a field of Alfvén waves, including waves at the right wavelength λ to resonate with particles at some momentum p , and some gyroradius $r_g(p)$. Let the waves at this wavelength have amplitude B_1 , and energy density $U_{res} = B_1^2/8\pi$. Their energy will be small compared to the mean field energy, $U_{Bo} = B_o^2/8\pi$. Thus the deviation of the field lines in the wave, from the mean field direction, is $\delta\phi \sim B_1/B_o$. We can describe the effect of the waves on the particles by noting that, when a resonant particle stays in phase with

a wave over one wave period, its pitch angle will have changed by $\sim \delta\phi$ compared to its initial pitch angle. However, this is a random process; after N encounters with Alfvén waves, the particle’s net change in pitch angle is $\phi = \sqrt{N}\delta\phi$. Thus, the waves act as scattering centers. We can define a scattering mean free path as the distance over which a particle’s pitch angle changes by $\phi \sim 1$ rad:

$$\lambda_{mfp} = N\lambda \simeq \left(\frac{B_o}{B_1}\right)^2 r_g = \frac{U_{Bo}}{U_{res}} r_g \quad (8.12)$$

Thus: if we know the energy density of *resonant* waves, we can find the mean free path, and thus the collision time, for the particles to scatter on the waves. This is the mechanism by which relativistic particles are tied to the galactic system; and it is probably important in the acceleration of the particles to these high energies (a version of this will appear in the homework).

Wave growth. When Alfvén waves exist at the right $\lambda_{res}(\gamma)$, then, particles with energies satisfying (8.811) will be strongly tied to the background plasma. We should note that there are a couple of ways in which Alfvén waves can be generated. First, as mentioned above, any disturbance to the background plasma will generate Alfvén and sound waves. Wave sources could include stellar winds, cloud motions, novae and supernovae, stellar random velocities, etc. In addition, it turns out that the resonant particles themselves can generate the waves. It turns out that particles with (i) an anisotropic velocity distribution, such as in a directed beam, and (ii) streaming speed $v \gg v_A$ will generate Alfvén waves,² at wavelengths given by (8.11). These self-generated waves will then scatter and isotropize the particles, reducing their streaming speed to $\sim v_A$.

Energy limits for wave-particle interaction. What are the limits on particle energies that can be scattered and accelerated by Alfvén waves? The upper limit is given by the maximum wavelength that can exist in the system; this can’t be larger, clearly, than the scale size of the system. For the lower limit, it turns out that Alfvén waves can only exist for frequencies $\omega < \Omega_p = eB/m_p c$, the *subrelativistic* proton gyrofrequency. (At higher frequencies, the proton response complicates the wave behavior, and the wave

²Think back to chapter 7 and the two-stream (beam) instability, in which a plasma “beam” trying to penetrate another plasma generates turbulent plasma waves. The detailed physics is different here and there, but the general picture is the same – a beam/plasma system and a resonant interaction.

changes nature). Thus, there is a maximum wavenumber $k_{max} = \Omega_p/v_A$, and a minimum resonant wavelength, $\lambda_{min} = 2\pi/k_{max}$, which can exist for Alfvén waves. From (8.10), we see that there is a minimum particle energy which can “see” Alfvén waves. When one works out the details of the algebra, it turns out for protons,

$$\gamma_{min,p} = 1 + \frac{2v_A^2}{c^2\mu^2} \quad (8.13)$$

where μ is the cosine of the particle’s pitch angle. For electrons, the equation is best solved in limiting cases. In a high-density plasma, with $v_A^2 \ll (m_e^2/m_p^2) c^2\mu^2$, the minimum electron energy that satisfies (8.7) is

$$\gamma_{min,e} = 1 + \frac{v_A^2}{c^2\mu^2} \frac{m_p}{m_e} \quad (8.14)$$

In the opposite limit, for a low-density plasma, the minimum energy for resonance becomes

$$\gamma_{min,e} = \frac{m_p v_A}{m_e c\mu} \quad (8.15)$$

Thus, the acceleration of low energy (say, thermal) particles by any mechanism that depends on resonant Alfvén waves is quite different for electrons and protons. A significant number of thermal protons can resonate with Alfvén waves, and can (in principle) be accelerated by them. But thermal electrons have less chance of acceleration; in a low density plasma, especially, they must already have $\gamma \gg 1$ to resonate with the waves.

8.3 Magnetic Buoyancy

Magnetic fields can affect buoyant behavior in various ways. Several situations are possible, involving simple buoyant bubbles, or buoyant flux tubes. Applications range from solar prominences, to the gaseous halo of our galaxy, to the structure of some radio galaxies. You’ve already worked with the buoyant rise speed (in previous homework). Here we need to think about whether or not an equilibrium (such as a hydrostatic atmosphere, or the galactic disk) is buoyantly unstable.

8.3.1 Convective stability (unmagnetized)

To set the stage, we first need to look at the problem without magnetic effects. Following Shore³, let’s think about a duck.

³An *Introduction to Astrophysical Hydrodynamics* (Academic Press) 1992, ch. 9.

Picture a duck sitting calmly on a pond. . . If we say that the bird is buoyant, we mean that if we depress him a bit by pushing from above, he will bob back to the surface and, ignoring his agitation, bounce up and down for awhile. This is called *neutral bouyancy*. If, on the other hand, our duck is not well preened and therefore is not waterproof, and we push down on him, he may sink. Now, think of a blob which is hotter than its surroundings. It will begin to rise, since we already know that its density will be lower than that of the surrounding medium, and it will thus be bouyant. If it remains underdense, it will continue to rise – we call this an *instability*. It will continue to rise until it reaches a level at which it is neutrally bouyant again. On the other hand, if the blob is pushed down, and if it remains overdense, it will sink until it reaches a point at which the density again allows for stable balance.

Now, move on to a more relevant example, namely an unmagnetized, hydrostatic atmosphere. Think about a blob (“parcel”) in this atmosphere: assume it starts at some vertical position z , with density and pressure in balance with its surroundings (as in Figure 3). Thus: it starts at $\rho_{in} = \rho_{out} = \rho_1$ and $T_{in} = T_{out} = T_1$. Now, raise it some distance dz , and assume it evolves *adiabatically*. Thus, it reaches a new density and temperature,

$$\rho_{in}^* = \rho_1 + \left(\frac{d\rho}{dz}\right)_{ad} dz ; T_{in}^* = T_1 + \left(\frac{dT}{dz}\right)_{ad} dz \quad (8.16)$$

The surroundings, however, are not necessarily adiabatic: they have some other dT/dz and $d\rho/dz$ values (specified by the situation – for instance the heating/cooling balance for the outer layers of a star).

But now: the blob will be unstable if, when it has risen this δz , it is at a *lower* density than the surrounding atmosphere – in that case it will keep rising. Thus, our condition for instability is just a condition on the external density and temperature gradients. Therefore, the atmosphere is buoyantly unstable if $(d\rho/dz)_{ad} < (d\rho/dz)_{atm}$. Because we usually consider situations with $d\rho/dz < 0$, for instance a hydrostatic atmosphere, the condition for *instabilty* is often written in terms of absolute values:

$$\left|\frac{d\rho}{dz}\right|_{ad} > \left|\frac{d\rho}{dz}\right|_{atm} : \left|\frac{dT}{dz}\right|_{ad} < \left|\frac{dT}{dz}\right|_{atm} \quad (8.17)$$

Thus: if the outside (atmospheric) temperature changes too rapidly with altitude, the atmosphere is

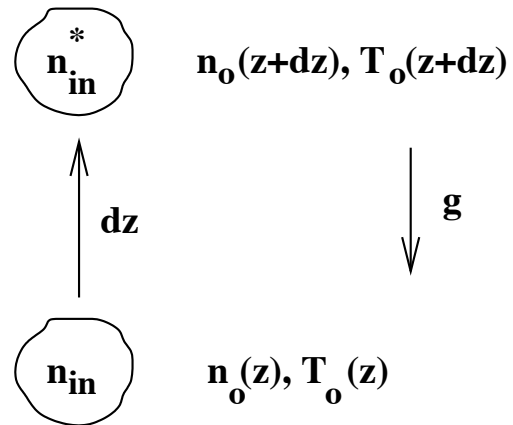


Figure 8.3 Setting the stage for the bouyant (in)stability. A blob starts at height z , in a local gravitational field g . The blob is initially at the same pressure and density as its surroundings: $n_{in} = n_o, T_{in} = T_o$. It rises slowly, staying in pressure balance with outside; it expands adiabatically as it rises, going to some n_{in}^* . The question is, how does its new density compare to the density outside?

convectively unstable. An underdense blob will continue to rise, and an overdense blob will sink.

8.3.2 Bouyant instability, magnetized

How will a B field change things? To think about it, let’s replace our blob with a more realistic geometry, such as a flux tube rising from the surface of the sun (as in Figure 4). We will find another condition is necessary for instability (in addition to the structure of the surrounding atmosphere). In this geometry, instability requires overcoming magnetic tension (holding the ends of the flux rope down) as well as simple convective instability. To see this, consider an isolated flux tube, such as might give rise to a sunspot. Let the external gas have a density scale height $H = k_B T / m g$ (refer back to earlier work for hydrostatic equilibrium with no magnetic field). If the magnetic field is confined in the flux tube, (no B field outside), and the intial state is in pressure balance, we can again write internal-external pressure balance as $p_i + B^2 / 8\pi = p_e$.

Assuming the gas inside is at the same temperature, it must be at lower density than the outside. Thus leads to a bouyancy force, as you found before:

$$F_{buoy} = g(\rho_e - \rho_i) = g\Delta\rho = \frac{B^2}{8\pi H} \quad (8.18)$$

Say, now, that the flux tube is bent upwards, locally, with a radius of curvature R . If R is short, magnetic tension will pull the tube back towards its initial position, giving a stable system. If, however, R is long, the

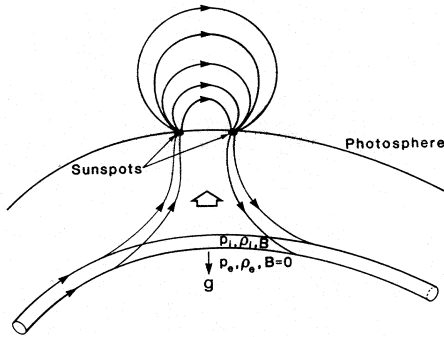


Figure 8.4 The geometry of a sub-surface flux tube before it erupts from the sun due to bouyancy, and its possible post-eruption state; from Tajima & Shibata figure 3.17.

buoyancy force will overcome the tension, leading to instability. Comparing these two forces, we find instability occurs if

$$g\Delta\rho > \frac{B^2}{4\pi R} \quad (8.19)$$

and thus the flux rope is unstable if $R > 2H$.

8.3.3 Parker instability

Now, let's apply this to a different geometry, a 1D planar system (such as the ISM in the galaxy). In this context magnetic bouyancy is referred to as the *Parker instability*. Let the magnetic field be horizontal, and fully mixed with the gas. Picking \hat{z} as the vertical direction, that means we take $\mathbf{B} = B_y(z)\hat{y}$; and describe the gas by density $\rho(z)$, pressure and sound speed $p(z) = c_s^2\rho(z)$, and gravitational field \mathbf{g} .

We need a model for the unperturbed state. The simplest assumption we can make is that the ratio of gas to magnetic pressure is constant at all altitudes: $B^2/8\pi p = \alpha_o = \text{constant}$. Magnetostatic balance, then, can be written

$$(1 + \alpha_o)\frac{dp}{dz} = -\frac{p}{c_s^2}g \quad (8.20)$$

This again gives us a simple exponential atmosphere. The difference here is that the scale height involves the magnetic as well as thermal pressures:

$$\frac{p}{p_o} = \frac{\rho}{\rho_o} = \frac{B^2}{B_o^2} = e^{-z/\Lambda} \quad (8.21)$$

where $\Lambda = (c_s^2 + v_A^2/2)/g$ is the new scale height. (Check: yes this is larger than the unmagnetized H – which makes sense, due to the extra pressure suport of the magnetic field).

Now, consider a small perturbation: let a horizontal layer (or flux tube) be raised some small Δz . The plasma in this layer can flow freely along the field lines – thus it will “slide down to the bottom”, leaving the top part of the perturbation at a lower density than its surroundings. If the perturbation has horizontal scale λ , simple geometry gives us its radius of curvature: $r \simeq \lambda^2/8\Delta z$. To check the stability of this perturbation, we again compare the bouyant forces to the restoring force, magnetic tension. Let “o” refer to the altitude of the undisturbed sheet. The density in the gas at the top of the perturbation does not know about the B field, because the gas can slide down along the field lines. So if the perturbation is taken slowly, to reach a new quasi-hydrostsatic balance “inside”, the density inside is

$$\rho_i(\Delta z) \simeq \rho_o e^{-\Delta z/H} \simeq \rho_o \left(1 - \frac{\Delta z}{H}\right) \quad (8.22)$$

The external density does know about magnetic pressure, however:

$$\rho_e(\Delta z) \simeq \rho_o e^{-\Delta z/\Lambda} \simeq \rho_o \left(1 - \frac{\Delta z}{\Lambda}\right) \quad (8.23)$$

Thus, the difference between the two scale heights drives the instability in this case. If we again compare bouyancy to the restoring force of magnetic tension, we find the instability condition for this case:

$$\lambda^2 > \frac{16\Lambda^2\alpha_o}{(1 + \alpha_o)^2} \quad (8.24)$$

where α_o is still the ratio of gas to magnetic pressure.

Key points

- Alfven waves, magnetosonic waves; what they are, how they work (reprise);
- Cosmic rays – basic picture, in the galactic setting;
- Resonant interaction between CR and Alfven waves;
- Buoyant instability, non-magnetized;
- Magnetic bouyancy, Parker instability.