9 Magnetic Topology: Dynamos and Reconnection

Magnetic flux freezing is a very good approximation in most astrophysical environments (and is a very handy tool). When flux freezing holds, the *topology* of the magnetic field lines remains constant.[1] We know, however, that flux freezing can be violated on small scales, where resitivity becomes important (such as a thin current sheet). It follows that the topology of the field is no longer invariant – the field lines can "break and reconnect" (well, sort of) in resistive regions.

There are two particularly important applications of resistive MHD: magnetic reconnection and dynamo theory.

9.1 Magnetic Reconnection

In these notes I'm emphasizing the details of simple, 2D reconnection. For context, note that reconnection provides one method of heating a magnetized plasma.

I just argued that magnetic topology must be preserved in the absence of resistivity. But is the converse obvious? To illustrate how resistivity can "break" and "reconnect" field lines, think about the geometry in Figure 9.1. We know resistivity is important in regions of high current density - such as the central region (around OP). If we set up this geometry and waited awhile, the central magnetic field would decay as the current layer supporting them is dissipated. This would deplete the magnetic pressure in this region. The plasma above and below this region would be pushed inwards by its own pressure, bringing a in a fresh supply of field (and plasma).

Now, look at this in more detail (I'm following Choudhuri's discussion here). The field lines ABCD and A'B'C'D' move inwards, with velocity v_{in}. Eventually the BC and B'C' parts of the field lines decay away. The AB part of the field line is moved to EO, and the A'B' part of that field line is moved to E'O. Thus, these "fragments" of two original field lines now make up one new field line, EOE'. And similarly, the parts CD and C'D' eventually make up a new field line, FPF'. Thus, "cutting and pasting" of field lines (otherwise known as reconnection) takes place in the central region. And, of course, there must be plasma flow away from the region – sideways in this cartoon – to conserve mass.

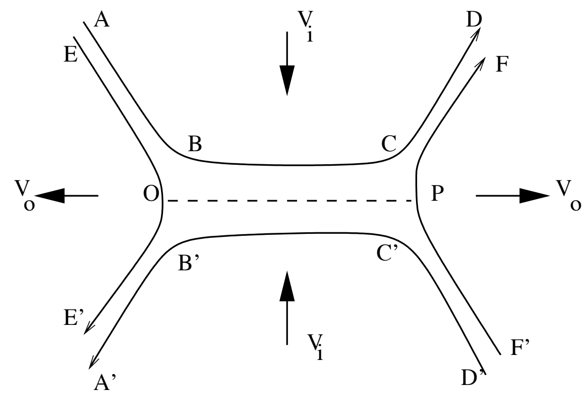

Figure 9.1 Illustrating a sinple reconneciton geometry; see text for discussion. Following Choudhuri figure 15.2.

We can be more quantitative about this geometry, and find simple scaling laws to describe this situation. The model I'm describing here is *Sweet-Parker reconnection* — it's illustrated in Figure 9.2.

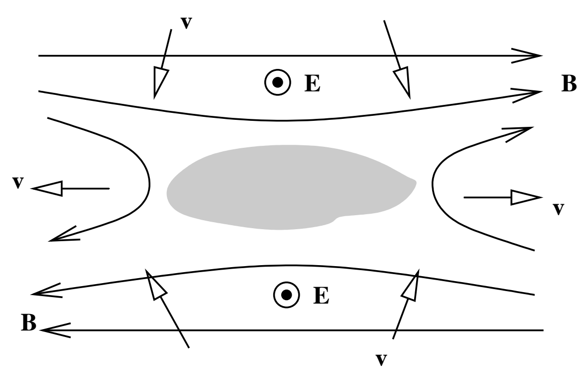

Figure 9.2 Geometry of Sweet-Parker reconnection. The current sheet (grey shaded area) has thickness l and lateral extent L The input velocity is v_{in} and the output velocity is v_{out}. Quantities far away from the current sheet are labelled with subscript o. Following Cowley figure 5.5.

First, assume the flow is incompressible – that it stays at constant density (which turns out to be a good approximation if v_{in} and v_{out} are both subsonic. Mass conservation then requires

$$v_{in}L = v_{out}l \tag{9.1}$$

Next, consider force balances. In the vertical direction, we note that $B \to 0$ at the center of the current sheet, and that $p \to p_{max}$ there (its maximum value). Pressure balance in this direction therefore requires

$$\frac{B_o^2}{8\pi} \simeq p_{max} - p_o \tag{9.2}$$

[1] Why is this? You can think about the field lines as being tied to the fluid; they move and stretch as the fluid moves, but cannot cross each other (how could they do that without "untying" themselved from the flow?)

Along and within the current sheet (call that the $\hat{\mathbf{y}}$ direction), there is no $\mathbf{v} \times \mathbf{B}$ force, so only the pressure gradient accelerates the flow. We have then,

$$\rho v_y \frac{\partial v_y}{\partial y} \simeq -\frac{\partial p}{\partial y}\ ; \quad \rho \frac{v_{out}^2}{2} \simeq p_{max} - p_o \qquad (9.3)$$

Thus, the outflow speed must be

$$v_{out}^2 \simeq v_A^2 = \frac{B_o^2}{4\pi\rho} \qquad (9.4)$$

Now, what about the inflow speed v_{in}? Clearly, by mass conservation, it must be $v_{out} \simeq v_{in} L/l$; but what sets l? We can try two different arguments here.

1. First, go back to the induction equation (5.11). In a steady state, it is $\nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} = 0$, which gives (by dimensional/scaling analysis)n

$$\frac{v_{in} B}{l} \simeq \frac{\eta B}{l^2}\ ; \quad v_{in} \simeq \frac{\eta}{l} \qquad (9.5)$$

This tells us that we can keep the diffusion region steady if the rate at which flux is brought in is equal to the rate at which it is annihilated.

2. Alternatively, note that there must be an $\mathbf{E}$ field "out of the page", as shown in figure 9.2, to maintain the current sheet. From Maxwell, we get

$$\mathbf{j} = \sigma \mathbf{E}\ ; \ \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j} \ \Leftrightarrow \ \frac{B}{l} \simeq \frac{4\pi}{c}\sigma E \qquad (9.6)$$

where σ is the electrical conductivity. But now, the incoming plasma charges must $\mathbf{E} \times \mathbf{B}$ drift[2] into the region: thus

$$v_{in} = c\frac{E}{B} \simeq \frac{c^2}{4\pi\sigma}\frac{1}{l} = \frac{\eta}{l} \qquad (9.7)$$

(in that last step I've used the definition of the magnetic resistivitiy, $\eta = c^2/4\pi\sigma$, from chapter 5). And look: this agrees with (9.5)!

OK: now, combine our answer (9.5 or 9.7) with (9.1), and we get our result, the inflow velocity and thickness of the dissipation layer:

$$v_{in}^2 \simeq \frac{v_A \eta}{L}\ ; \qquad l^2 \simeq \frac{\eta L}{v_A} \qquad (9.8)$$

This gives us, finally, the *spontaneous reconnection rate*; the rate of slow inflow that allows things to go steadily. This is indeed slow – in many situations (for instance solar flares), the plasma conductivity is high (given by the Coulomb collision value), so that η is low; and the reconnection timescale ($\simeq L/v_{in}$) is much to long to explain the observations. People have, therfore, spent a lot of time trying to invent faster versions of this model. Some are as follows ..

[2]Check back in Chapter 2: $\mathbf{v}_E = c(\mathbf{E} \times \mathbf{B})/B^2$, from (2.17).

• **Petschek reconnection** has gotten a lot of attention in the literature. The idea was that internal structures in the flow (shocks toward the edges of the current sheet) would affect the velocity field and narrow the width of the sheet, to $L \sim l$. This would of course make the inflow rate nearly independent of η. The last I heard is that lab experiements (*cf.*review by Kulsrud, 1998) were not confirming this model.

• **Compressible flow**. Another suggestion is that the plasma in the dissipation region (DR) is compressed. If this is the case, then (9.1) is replaced by $v_{in}\rho_o L \simeq v_{out}\rho_{DR} v_{out}$. This will increase v_{in} relative to v_{out} – its a plausible idea, but I'm not aware of any lab tests yet.

• **Anomalous diffusion**. This is my personal favorite, and is a good example of how plasma microphysics can be important in macroscopic situations. That is: is the usual value of η, based on Coulomb collisions in the plasma, the right value? The answer is almost surely that it is not. If the current density is high enough, we have a "two-stream"-like situation in the current sheet, and can expect plasma turbulence to be generated.

But can this be quantified? One commonly used estimate is as follows. Recall the microscopic origin of plasma conductivity, from chapter 3:

$$\sigma = \frac{j}{E} =\simeq \frac{ne^2}{m}\tau_{coll}$$

But now, what is τ_{coll} when we're working with plasma turbulence? A common guesstimate, which is thought to be an upper limit to the resistivity (lower limit to τ_{coll}) is to set $\tau_{coll} = 2\pi/\omega_p$, if $\omega_p = \left(4\pi n e^2/m\right)^{1/2}$ is the usual plasma frequency. This gives an estimate of the *anomalous resistivity*.

9.2 Reconnection: other approaches

Reconnection is a very active field these days; the simple 2D models we've just seen are probably way too simple. I'll just store a few comments here, thinking about what interests me most.

9.2.1 Non-steady reconnection

Here is my personal impression: who says steady-state reconnection is relevant to any natural situation? That is: the arguments above show that steady reconnection models must be forced, sometimes rather severely, to connect with what we think is occuring in nature. But examples of non-steady reconnection events are easy to find:

• Reconnection in solar flares. Flares are seriously transient events; the large amount of energy released is believed to be due to very fast annihilation of magnetic field, in a reconnection event. Flares have motivated much of the work on steady-state models; however as an outsider I suspect time-dependent, patchy, localized reconnection must be taking place.

• Reconnection at the magnetopause, where the solar wind hits the earth's magnetic field. Spacecraft observations suggest this is very patchy, localized and time-dependent. Picture, for instance, magnetic flux tubes being carried along in the solar wind; and let one of them impact the magnetopause, which also has its field bunched into ropes. This process can allow solar wind plasma, and field, to penetrate into the magnetosphere.

9.2.2 Driven reconnection

It's worth remembering that the 2D models above are all "spontaneous": put two misaligned $\mathbf{B}$ fields together and wait to see how quickly they reconnect. But nature does not always work this way. One can envision a situation in which the two anti-parallel magnetic structures are *driven together*, by large-scale flows in the system (one example of this is MHD turbulence, in which different parts of the plasma move in random directions, at a speed set by the energetics of the turbulence). In this case one (this one at least) expects that the inflow speed (v_{in}) in our notation above) will be set by the large-scale flow (say the turbulent speed). How can the reconnection site adjust to this? Some authors (*e.g.* Parker) suggest that the internal structure of the reconnection layer – its thickness, density or resistivity (set by microscale turbulence therein) – will adjust as necessary.

9.2.3 Three-dimensional reconnection

Another observation: only rarely can a reconnection event be well described by a two-dimensional analysis. My last example in fact assumed this – because the intersection of two magnetic flux ropes is clearly a three-dimensional process. Going to 3D is challenging, and work is only starting here (helped significantly by numerical simulations).

9.3 MHD Dynamos

Where do magnetic fields come from? In the lab, the answer is easy: "currents". In magnetic solids, the currents are those of well-ordered electrons spins in ferromagnetism. More typically, currents in the lab — and their consequent $\mathbf{B}$ fields — come from obvious things like batteries and wires. The issue is then, what drives the currents? My dictionary defines a dynamo as

> "a device for converting mechanical energy into electrical energy, usually by expending the mechanical energy in producing a periodic motion of a conductor and a surrounding magnetic field".

A simple lab version of this is called the *unipolar dynamo*, in Figure 9.3. This involves a conducting disk, threaded by a $\mathbf{B}$ field, which rotates about its axis. This induces a radial $\mathbf{E}$ field, $\mathbf{v} \times \mathbf{B}/c$, and thus a potential drop between the axis and the edge of the disk. If you hook up wires in the right way you'll have a current – and this current will create its own magnetic field.

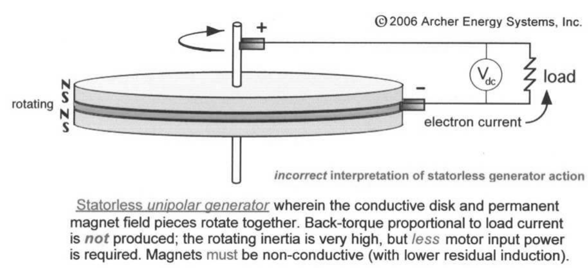

Figure 9.3 A simple unipolar dynamo (in a less than simple figure from www.stardrivedevice.com, the best figure I could find). The conducting disk moves through an (externally supported) $\mathbf{B}$ field as it rotates about its axis. The resultant EMF supports a potential drop between the axis and edge of the disk — which can drive a current.

What about astrophysical magnetic fields? To be specific, what is the origin of the earth's field, or the sun's field? It's easy to think of what doesn't work. One, even solid planets like the earth can't be ferromagnetic (because the core temperature is well above the Curie temperature at which permanent magnetism disappears); and clearly stars and galaxies can't be ferromagnetic at all. Two, we can't assume the fields are primordial — were somehow created when the sun/earth/galaxy formed — because we know the resistivity of the plasmas in question, and thus we know how long it would take a primordial current to dissipate. Such calculations predict that primordial fields would long ago have died away; but we know that stars, planets, and galaxies are still magnetized.[3]

Thus, we still must ask, "What supports astrophysical $\mathbf{B}$ fields?" The answer is still currents, but what drives astrophysical currents? We can't expect a device such as in Figure 9.3 exists inside a planet, or star, or whatnot ... so we need to find a way to drive *fluid motions* which can maintain the $\mathbf{B}$ fields we observe. This question gets us into what's called *dynamo theory*. To approach this, go back (yet again) to the induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \tag{9.9}$$

We know the second term describes resistive decay; if we're lucky the first term can be a *growth* term. The first term describes magnetic induction, when $\mathbf{v} \times \mathbf{B}$ creates a local $\mathbf{E}$ field, and thus an EMF, which can drive a current. If the geometry is right, this current can make the initial (seed) magnetic field grow — giving us a *dynamo*. But the devil is in the details – how can the right flow field be created and maintained naturally?

9.3.1 Cowling's theorem

We can start by seeing what *won't* work. That is, most astrophysical models assume simple, symmetric geometries; but these can't support a dynamo.

To be specific, we need to prove Cowling's theorem: *it is not possible to maintain a steady dynamo in an axisymmetric system*. To do this, I follow Cowling's original (1934) argument, as presented by Choudhuri. Start by assuming we do have an axisymmetric dynamo: one with $\partial/\partial t = \partial/\partial \phi = 0$. Consider a plane through the symmetry axis: the projections of the field lines on this plane must be closed curves (think of a simple magnetic dipole). There will be at least one neutral point in this plane (a point where the closed field lines center) – and j_ϕ must be non-zero here, while $\mathbf{B}$ has only a ϕ component at this point. Take a line integral of Ohm's

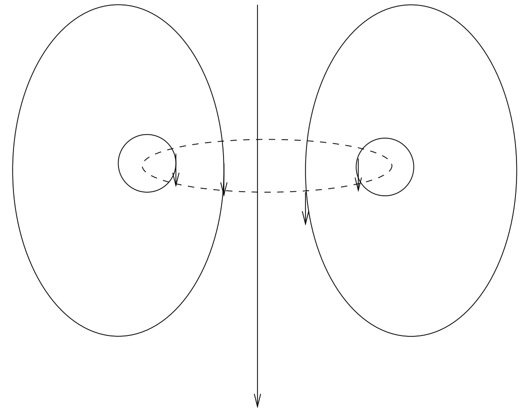

Figure 9.4 Illustration of geometry for Cowling's theorem. Following Choudhuri Figure 16.3.

law (e.g. equation 5.10) along a closed loop through these neutral points, enclosing the symmetry axis:

$$\frac{1}{\sigma} \oint j_\phi dl = \oint \mathbf{E} \cdot d\mathbf{l} + \oint \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l} \tag{9.10}$$

But now: the second term vanishes, because $\mathbf{B} \parallel d\mathbf{l}$ if this loop goes entirely through neutral points. The first term vanishes, because

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int \nabla \times \mathbf{E} \cdot d\mathbf{S} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

and this last is zero by our steady-state assumption. However, the LHS of (9.10) is non-zero, as j_ϕ is finite. Thus, we have a contradiction; and Cowling's theorem is proved.

It follows, then, that we must relax the assumption of axisymmetry; and yet we want to maintain the large-scale axisymmetry which we know describes objects like the sun, the earth, or the galaxy. The answer is to introduce small-scale asymmetries — best done, astrophysically, with small-scale disordered fluid motion, such as convection or turbulence.

9.3.2 Parker's solar dynamo

The classic dynamo model is due to Parker (1955), and is meant to describe the solar magnetic field. It is best presented qualitatively — refer to Figure 9.5 for the cartoon.

Say the solar field starts mainly dipolar (this is roughly consistent with observations of the global field, just above the solar surface). The sun does not rotate as a solid body; near the surface, the equator rotates faster than the polar regions. This will stretch our dipolar

[3] In addition, we know that the sun's field reverses pretty regularly, every 11 years or so; and the earth's field reverses less regularly, every $10^4 - 10^5$ years. This clearly requires some internal, self-governing mechanism.

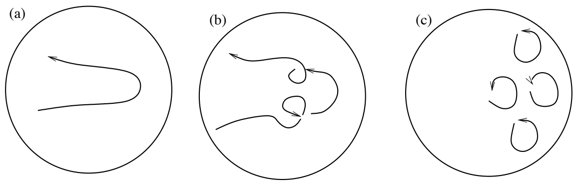

Figure 9.5 Parker's model of the solar dynamo, at the cartoon level. (a) Differential rotation (the sun's equator rotates faster than the poles) stretches initially dipolar field in the toroidal direction. (b) Coriolis forces acting on surface convective cells generates local poloidal fields. (c) The opposite sense of the Coriolis force in the north and south hemispheres, combined with the opposite sense of the initially toroidal field, results in a strong net poloidal field, rather than a randomly directed set of field loops. (These loops are shown projected in the meridional plane.) Following Choudhuri, Figures 16.4.

field, generating toroidal components. Thus, it is no problem to generate toroidal field if the body has differential rotation. But this cannot be all of the story. Such a stretched toroidal field will have many local field-line reversals, and if nothing else happens it will simply decay away due to resistive dissipation.

However, the upper layers of the sun are convectively unstable. In this region, plasma blobs rise and fall.[4] Now, these vertically moving blobs are subject to a Coriolis force, due to the sun's overall rotation. The blobs therefore rotate as they rise; they act like little cyclones, and formally we say that their their motion has a net *helicity* (that means the small-scale motions do not have mirror symmetry: for instance a flow with $\mathbf{v} \cdot (\nabla \times \mathbf{v}) \neq 0$ is helical). Look at (b) of Figure 9.5: this cyclonic motion twists the magnetic field back into poloidal loops. Remember that both the direction of B_ϕ and of the Coriolis rotation are opposite in the north and south hemispheres: this means the direction of the poloidal field component generated is the *same* in the two hemispheres. We therefore have a fully working dynamo: poloidal fields are generated by the helical convective (turbulent) motions, while toroidal fields are generated by differential rotation. The whole system must be stabilized by dissipation – that is resistivity will keep each field component from getting too large.

[4] In chapter 8 we talked about (in)stability to buoyancy – an unstable atmosphere will develop strong convection.

9.3.3 Scale separation and turbulent dynamos

We argued "by cartoon" that helical, convective motions on the sun (or the earth) can maintain the large-scale $\mathbf{B}$ field. That is, we're arguing that *small-scale* turbulent motions can add up to a net *large-scale* dynamo.

To get a sense of how this works, and what's needed to make it work, we need to be a bit formal. Split the velocity and magnetic fields into mean and fluctuating parts:

$$\mathbf{B} = \mathbf{B} + \mathbf{b}\,; \qquad \mathbf{v} = \mathbf{V} + \mathbf{v} \tag{9.11}$$

where we're assuming that $\mathbf{b}$ and $\mathbf{v}$ have zero mean,, and also that they are small-scale – that they vary over much smaller spatial scales than $\mathbf{V}$ and $\mathbf{B}$ do. What effect do they have on the induction equation (5.11, also 9.9)? Let's split it into large-scale (mean) and small-scale (fluctuating) parts. For the mean field, we get

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \nabla \times \boldsymbol{\varepsilon} + \eta \nabla^2 \mathbf{B} \tag{9.12}$$

where the important new term is

$$\boldsymbol{\varepsilon} = -\langle \mathbf{v} \times \mathbf{b} \rangle \tag{9.13}$$

This describes the net EMF due to the fluctuating $\mathbf{v}$ and $\mathbf{b}$.

But now, we must ask whether $\boldsymbol{\varepsilon}$ has any interesting large-scale effect. If $\mathbf{v}$ and $\mathbf{b}$ are rapidly varying, have zero mean, and uncorrelated, we'd expect the mean of their product to be zero. It turns out ("can be shown") that things are interesting (non-zero) if the turbulence satisfies two conditions: (i) it must be helical, satisfying $\mathbf{v} \cdot \nabla \times \mathbf{b} \neq 0$; and (ii) it must be resistive; $\eta \neq 0$. If both of these conditions are met, the turbulent EMF, $\boldsymbol{\varepsilon}$, will be non-zero *on large scales*. In particular, it may be the case that $\langle \mathbf{v} \times \mathbf{b} \rangle = \alpha \mathbf{B}$ (*i.e.*, that $\boldsymbol{\varepsilon}$ has a component along $\mathbf{B}$). If this is so, then we have an effective dynamo term:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{B}) + (\text{other stuff}) \tag{9.14}$$

If this works – if $\boldsymbol{\varepsilon} = \alpha \mathbf{B}$ — then we can see two useful astrophysical consequences. One is balancing ohmic losses, as in the sun or the earth — and (in principle) accounting for the occasional field reversals in each body. The second is "growing" the $\mathbf{B}$ field in the first place. To see this, note that (9.14) allows solutions $\mathbf{B} \propto e^{\alpha t}$, if α is constant in time. That's a growing $\mathbf{B}$ field, with growth time $\sim L/\alpha$ (some large-scale length scale L). We might expect that a small seed field would grow exponentially until some other physics (dissipation? back reaction on the driving fluid?) comes into play.

9.3.4 Astrophysical dynamos in the lab

Finally, a few words about trying to do this in the lab. Everything above is still pure theory — it would be good to verify directly that an $\alpha\omega$ dynamo (rotation plus turbulence, as in the sun), or an α^2 dynamo (pure turbulence) can really make a large-scale ordered $\mathbf{B}$ field. Several groups are working on this, including NMT's very own Stirling Colgate. The experiments use liquid metal — usually liquid sodium — in some sort of rotating system (the ω in an $\alpha\omega$ dynamo), and try various ways to induce turbulence (the α) in the flow. The last I heard, no one had sucessfully made their dynamo work — but I think the field's progressing. Check the Feb 2006 issue of *Physics Today* if you'd like more details.

Key points

• Reconnection – simple 2D model; "what is a reconnection rate?"

• Reconnection – extensions of the 2D model (anomalous effects, non-steady, driven, etc).

• Dynamos – what they are; what they need (helicity). Parker's model for the sun.

• Dynamos — why turbulence matters; $\alpha\omega$ and α^2.