10 Pair plasmas in Astrophysics

Electron-positron pairs can also be important in the physics of high energy plasmas and their interaction with radiation. To set the stage: recall that the electron rest mass energy is 511 keV (or a temperature of 6×10^9 K).

NOTATION: in this chapter, $\epsilon = h\nu/m_e c^2$ is the *normalized* photon energy (relative to the electron rest mass). The normalized lepton energy is $\gamma = E/m_e c^2$, as usual. Watch out: γ is also the "reaction notation" for a photon (as in eqn. 10.1).

10.1 Pair annihilation

Free positrons will, of course, annihilate on electrons (or any other form of "regular" matter). The most common decay is

$$e^+ + e^- \rightarrow \gamma + \gamma \tag{10.1}$$

The annihilation cross section, measured in the center of momentum (CM) frame, is

$$\sigma_{e^+e^-} = \frac{\pi r_o^2}{\gamma + 1} \left[\frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} \ln\left(\gamma + \sqrt{\gamma^2 - 1}\right) - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right] \tag{10.2}$$

Here, γ is the lepton energy (in the CM frame), and r_o is the classical electron radius, defined as $r_o = e^2/m_e c^2$ (in cgs). The Thomson cross section can be written $\sigma_T = 8\pi r_o^2/3$.

The cross section has two useful limits. In the low-energy case, $\beta \ll 1$, the cross section becomes

$$\sigma_{e^+e^-} \simeq \frac{1}{\beta} \pi r_o^2 \tag{10.3}$$

This shows that the annihilation probability is very high for electrons nearly at rest. The decay produces two photons very close to the rest energy of the leptons: $h\nu \sim m_e c^2$, or $\epsilon = h\nu/m_e c^2 \simeq 1$. This is the *annihilation line*. In the high-energy case, $\gamma \gg 1$, the cross section becomes

$$\sigma_{e^+e^-} \simeq \frac{\pi r_o^2}{\gamma} (\ln 2\gamma - 1) \tag{10.4}$$

The decay at high energies still produces two photons, but the photons have a much broader energy spread. The annihilation line becomes a broad annihilation spectrum.

It is worth remembering that the electrons and positrons will undergo all of the usual plasma interactions and radiation, during the time that they exist (before they annihilate). They can support plasma waves, emit synchrotron radiation, and all of the other processes we have studied.

10.2 Pair creation

Electron-positron pairs can be created in a wide variety of nuclear and electromagnetic interactions. I summarize the three mechanisms that seem most important astrophysically.

10.2.1 Two-photon pair production

Classical physics says that EM radiation is linear; superposition is allowed, without affecting either incoming signal. This is not always true, however. We learn from quantum physics that two photons can interact to produce pairs, or other particles, if their energy is high enough. The process of interest here is

$$\gamma + \gamma \rightarrow e^+ + e^- \tag{10.5}$$

It can occur if the total incoming photon energy, in the rest frame, is at least as large as two electron rest masses. In the lab frame, this translates to $\epsilon_1 \epsilon_2 \geq 1$ (I'm keeping photon energies normalized to $m_e c^2$, to shorten the notation. If you don't accept this limit, do the Lorentz transform for yourself, to check!). The kinematics are simple in the CM frame: the two incoming photons must each have the same energy, and likewise for the two created leptons. In the CM frame, let β be the lepton velocity, and $\gamma = (1 - \beta^2)^{-1}$ be its energy. Energetics require $\beta^2 = 1 - 1/\epsilon_1 \epsilon_2$. The cross section for this process is

$$\sigma_{\gamma\gamma} = \frac{\pi r_o^2}{2\gamma^2} \left[\beta(\beta^2 - 2) + (3 - \beta^4) \ln\left(\frac{1+\beta}{1-\beta}\right) \right] \tag{10.6}$$

This process can have an important impact on a luminous high-energy photon source (such as a gamma-ray burster, or a very hot accretion disk). Consider the optical depth of a gamma ray, at energy ϵ. It can react with any photon that has energy above $1/\epsilon$. If L is the luminosity of the source above this energy cutoff, then the optical depth of the γ ray photon[1] is

$$\tau_{\gamma\gamma} = \frac{L \sigma_{\gamma\gamma}}{4\pi R \epsilon m_e c^3} \tag{10.7}$$

[1] Can you derive this?

This shows that the source can be opaque to its own radiation, if it has high luminosity and small size. Such a source is called "compact".

It is also possible for pairs to be produced in photon-proton or photon-electron interactions: $\gamma + p \rightarrow p + e^+ + e^-$, and $\gamma + e \rightarrow e + e^+ + e^-$. These reactions have smaller cross sections than $\gamma\gamma$ pair production (smaller by about the fine structure constant), and seem to be less important astrophysically.

10.2.2 Pair production in pion decay

Another source of pair production is from pion decay. Pi and mu mesons are unstable particles; once created, they decay rapidly. The common decay chains are

$$\begin{aligned} \pi^{\pm} &\rightarrow \mu^{\pm} + \nu_{\mu}/\bar{\nu}_{\mu} \\ \pi^{o} &\rightarrow \gamma + \gamma \\ \mu^{\pm} &\rightarrow e^{\pm} + \nu_e/\bar{\nu}_e + \bar{\nu}_{\mu}/\nu_{\mu} \end{aligned} \tag{10.8}$$

Thus, neutral pions decay simply to γ-ray photons. Charged pions, however, decay to muons which in turn decay to leptons.

So: how are pions made astrophysically? We have seen one mechanism, the interation of cosmic-ray protons with the microwave background:

$$\gamma + p \rightarrow p + \pi + \cdots \tag{10.9}$$

This predicts that there should be some free pions, decaying to muons and leptons, everywhere in space.

In addition, particle-particle reactons can make pions. The most common (there are many more I'm not listing. . .) are proton-proton collisions:

$$\begin{aligned} p + p &\rightarrow p + n + \pi^{+} \\ &\rightarrow p + p + \pi^{o} \\ &\rightarrow d + \pi^{+} \end{aligned} \tag{10.10}$$

The velocity-weighted cross section for π production in a thermal pp reaction is

$$\langle \sigma_{pp} v \rangle \simeq 4 \times 10^{-16} \ln T_{12} \tag{10.11}$$

if T_{12} is the proton temperature in units of 10^{12} K. This form is good for temperatures above ~ 100 MeV; at lower temperatures the cross section drops rapidly. One application of this process is to hot accretion disks. Some models predict that the inner part of the disks will be hot enough for pion production to take place.

10.3 Magnetic pair production

Relativistic kinematics tells us that a free photon, in vacuum, cannot create a massive particle (or particles); the process cannot conserve energy and momentum. Single-photon pair production is possible, however, in the presence of a strong magnetic field. This process requires high photon energies (in order to create the lepton rest masses), and also high magnetic fields. The field must be close to the so-called critical field, given by

$$\hbar \frac{eB_{crit}}{m_e c} = m_e c^2 \; ; \;\; B_{crit} \simeq 4.4 \times 10^{13} \text{ G} \tag{10.12}$$

Energetics require that the photon satisfy

$$\epsilon \sin\theta \geq 2 \tag{10.13}$$

if θ is the angle between the photon's wavevector and the magnetic field. The probability of this process occuring is given in terms of an attenuation coefficient,

$$\kappa(\chi) \simeq 1.5 \frac{\alpha}{\lambda_c} \frac{B}{B_{crit}} e^{-4/3\chi} \; ; \;\; \chi = \frac{\epsilon}{2} \frac{B}{B_{crit}} \sin\theta \tag{10.14}$$

Here, $\alpha = e^2/\hbar c$ is the fine structure constant, and $\lambda_c = h/m_e c$ is the Compton wavelength of the electron. This quantity κ is essentially an absorption coefficient: the "opacity" the photon sees is $\tau = \int \kappa(\chi) dx$.

Magnetic pair production is thought to be important in pulsars. You recall that these are strongly magnetized neutron stars. The high electric fields induced by the rapid rotation and strong B field pull charges off the star's surface and accelerate them to very high energies. These charges then emit γ rays, either by curvature radiation (related to synchrotron emission) or by inverse Compton scattering of thermal X-rays emitted by the star. These γ rays can then pair produce. The pairs themselves emit more γ rays, probably through synchrotron radiation. These secondary γ rays then make more pairs, which emit more photons . . . and a *pair cascade* develops. The resultant dense pair atmosphere is thought to be the source of the coherent pulsar radiation.

A very similar process may occur close to a massive black hole in the center of an active galaxy. While these models have not been as extensively developed as pulsar models, the basic ingredients are the same. The black hole, or the accretion disk feeding it, are almost certainly magnetized. Rotation will induce strong electric fields, which can accelerate charges to high energies. In addition, the accretion flow is probably a strong γ ray source itself. Thus, both two-photon and single-photon pair cascades are possible in this setting. Such a pair plasma may be the initial content of the directed, relativistic radio jets created by the nuclear black holes.

Key points

• Pair annihiliation: what it is, the magnitude of the cross section.

• "Particle" pair production, two types, what it is.

• Magnetic pair production, what it is, its "probability".

11 (Inverse) Compton scattering

We have one more radiation mechanism to cover, which is particularly relevant to compact objects and relativistic (synchrotron) plasmas.

NOTATION: in this chapter $\epsilon = h\nu$ is the photon energy (in physical units, such as erg) ... sorry for the switch, folks, it's the standard notation.

11.1 Basic Tools

We need to use several different things here.

11.1.1 One event seen in the ERF

To start, go back to simple Compton scattering, as you saw it in modern physics. Work in a frame where the electron is at rest,[1] and hit it with an incoming photon, of energy ϵ'. The photon scatters through an angle θ_1', and to an energy ϵ_1', with

$$\epsilon_1' = \frac{\epsilon'}{1 + (\epsilon'/mc^2)(1 - \cos\theta_1')} \tag{11.1}$$

Clearly, the electron gains momentum and energy after the scattering, and the photon loses energy, $\epsilon_1' < \epsilon'$.

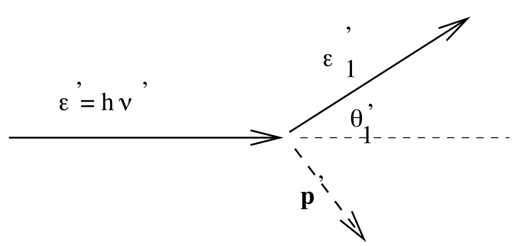

Figure 11.1 The geometry of compton scattering, in the electron rest frame (ERF). The photon comes in with $\epsilon' = h\nu'$, and leaves at angle θ_1' with ϵ_1'; the electron gains momentum $\mathbf{p}$.

11.1.2 Cross sections

In order to get the Compton-scattered spectrum, we'll need to know the probability that the electron scatters into angle θ_1' in the ERF, then average over all angles. The probability comes from quantum electrodynamics, and is called the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_o^2}{2}\left(\frac{\epsilon_1'}{\epsilon'}\right)^2\left(\frac{\epsilon'}{\epsilon_1'} + \frac{\epsilon_1'}{\epsilon'} - \sin^2\theta_1'\right) \tag{11.2}$$

where $r_o = e^2/m_e c^2$ is the classical electron radius, and $d\Omega$ is the differential solid angle for the scattering.

[1] This observing frame is called the Electron Rest Frame, ERF.

If we want the total scattering cross section, still in the ERF, we integrate (11.2) over $d\Omega$:

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$$

The general result is a long expression, and the limits are more useful. For $x = \epsilon/mc^2$:

$$\begin{aligned} x \ll 1: &\quad \sigma \simeq \sigma_T \ ; \\ x \gg 1: &\quad \sigma \simeq \frac{3}{8}\sigma_T \frac{1}{x}\left(\ln 2x + \frac{1}{2}\right) \end{aligned} \tag{11.3}$$

(recalling that $\sigma_T = 8\pi r_o^2/e$). Thus, for low photon energies, the scattering cross section is just σ_T; for high photon energies, it's more complicated. Note, finally, that these relations hold in the ERF. If we're looking at a scattering event in the lab frame, we know that the photon energy is $\epsilon' \simeq \gamma\epsilon$; thus the transition energy in (11.3), namely $x = 1$, becomes $\gamma\epsilon = mc^2$.

11.1.3 Remember your relativity

Here's a review if you need it.

• **Lorentz transforms.** Because our next step will be transforming between the ERF and the lab frame, we'll need to remember some Lorentz transforms. Figure 11.2 has the geometry. You remember that x position (along the motions) and time transform as

$$x' = \gamma(x - \beta ct) \ ; \ \ t' = \gamma(t - \beta x/c) \tag{11.4}$$

and, of course, the inverse transforms are just

$$x = \gamma(x' + \beta ct') \ ; \ \ t = \gamma(t' + \beta x'/c) \tag{11.5}$$

(Think about which way the "frame" is moving, and you can tell what to do about the $+$ and $-$ signs). You also probably remember that the coordinates y and z transverse to the motion don't change.

But now: several other important physical quantities transform just the same as position $\mathbf{x}$ and time do – these are called *4-vectors*. For a particle (massive or massless), momentum $\mathbf{p}$ and energy E are a 4-vector; so are wavenumber $\mathbf{k}$ and frequency ω if we're working with a wave. We therefore have (keeping track of units)

$$p_x' = \gamma(p_x - \beta E/c) \ ; \ \ E' = \gamma(E - \beta p_x c) \tag{11.6}$$

and

$$k_x' = \gamma(k_x - \beta\omega/c) \ ; \ \ \omega' = \gamma(\omega - \beta k_x c) \tag{11.7}$$

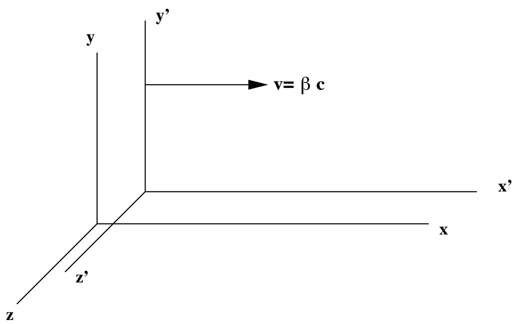

Figure 11.2 The geometry of our Lorentz transforms. The primed frame is moving at $v = \beta c$ along the x-axis; the unprimed frame is the "lab".

(and their respective inverses). Note that angle transforms can be found from the components of $\mathbf{p}$ or $\mathbf{k}$, as needed. If we apply these results to photons, which have $E = pc$ and $\omega = ck$, they collapse to one transform,

$$\omega' = \omega\gamma(1 - \beta\cos\theta) \tag{11.8}$$

where $\cos\theta = k_x/k$ selects out the $\mathbf{k}$ component along the direction of motion. We'll use this in a few minutes.

• **Invariants.** Some important quantities can be shown to be invariant – that is, to have the same value measured in either the lab frame or the moving frame. I'm not going to prove these here – you can look at Rybicki & Lightmann if you're curious. We'll need two important facts:

• We're interested in the power radiated (say by an accelerated particle). This is one invariant:

$$\frac{dE}{dt} = \frac{dE'}{dt'} \tag{11.9}$$

• We'll want to work with the photon spectrum; let $n(\epsilon)d\epsilon$ be the number of photons "at ϵ". It turns out that the ratio

$$\frac{n(\epsilon)d\epsilon}{\epsilon} = \frac{n'(\epsilon')d\epsilon'}{\epsilon'} \tag{11.10}$$

is also an invariant.

11.2 Scattering as seen in the lab

What then is "inverse" compton scattering? If the electron is moving, and has more (kinetic) energy than the photon, the photon will tend to gain energy in the collision (at the expense of the electron). The only difference between this and the simple version in (11.1) is the frame from which one views the collision ... but the moving-electron case is traditionally called Inverse Compton Scattering (ICS).

Thus, think of a situation where the photon has energy $\epsilon = h\nu$ and the electron has energy $\gamma m_e c^2$ before the scattering. We can analyze this by doing a Lorentz transform to a frame moving with the (γ, β) of the electron – the electron rest frame (ERF), and let it be the primed frame – in that frame (11.1) applies. In the ERF, we know the electron has $\beta' = 0$ (by definition!), and from (11.8), the photon has $\epsilon' = \gamma\epsilon(1 - \beta\cos\theta)$, where θ is the angle between the photon and electron momenta in the lab. Let the scattering take place, then transform back to the lab; the new photon energy in the lab is

$$\epsilon_1 = \epsilon_1'\gamma(1 + \beta\cos\theta_1') \tag{11.11}$$

(remember that ϵ', ϵ_1' are related by 11.1). Thus, for a given electron and photon energy before the scattering, ϵ' depends only on the scattering angle in the ERF.

11.2.1 Single particle radiation

The simplest application of this is to find the power emitted by an electron exposed to some photon field. By far the easiest way is to work in the ERF – where the scattering is simple – and transform to and from the lab as needed. We'll use basic Lorentz transforms, and the invariants (described above, in 11.9 and 11.10).

With this approach, it's not hard to find the total power scattered by our electron sitting in the radiation field $n(\epsilon)$. In the ERF, that power is

$$\frac{dE'}{dt'} = c\sigma_T \int n(\epsilon')\epsilon_1'(\epsilon', \theta_1')d\epsilon' \tag{11.12}$$

where we're assuming ϵ_1' from (11.1). The integrals here are over the photon spectrum. Because of the invariants, it's easy to write this in the lab frame:

$$\begin{aligned} \frac{dE}{dt} &= c\sigma_T \int (\epsilon')^2 \frac{n' d\epsilon'}{\epsilon'} \\ &= c\sigma_T\gamma^2 \int (1 - \beta\cos\theta)^2 \epsilon n(\epsilon) d\epsilon \end{aligned} \tag{11.13}$$

where we've used the basic Doppler shift, $\epsilon' = \gamma\epsilon(1 - \beta\cos\theta)$ in the last step.

Now: let the photon field be isotropic. The angle average of $(1 - \beta\cos\theta)^2$ is just $1 + \beta^2/3$; so the Compton-scattered power, angle-averaged and assuming an isotropic photon field, is

$$\frac{dE}{dt} = c\sigma_T\gamma^2 \left(1 + \frac{1}{3}\beta^2\right) u_{rad} \tag{11.14}$$

because $u_{rad} = \int \epsilon n(\epsilon) d\epsilon$ is the radiation density that the electron sees. Finally, then, we want the energy lost by the electron – its *Inverse Compton* power. That's just the scattered power minus the incoming power:

$$P_{IC} = \frac{dE}{dt} - c\sigma_T u_{rad} = \frac{4}{3} c\sigma_T \gamma^2 \beta^2 u_{rad} \quad (11.15)$$

(Does this form look familiar? Compare the relativistic limit, $\beta \simeq 1$, to the expression for synchrotron power, equation 9.5).

11.2.2 Single particle spectrum

Next, we want the spectrum of the scattered radiation. Unlike our previous applications (synchrotron and bremsstrahlung), we're not talking about Fourier analysis here. Rather, we know that a single scattering event leads to a photon energy ϵ_1, given by (11.11). But ϵ_1 depends only on the angles θ, θ_1'. One of these is the input condition (the angle between the incoming photons and the electron's motion). For the other, we know the probability of scattering into that angle – from $d\sigma/d\Omega$, (11.2). Thus, to get the photon spectrum – the probability of scattering giving an output ϵ' – we just can carry out the angle average of (11.11), using (11.2).

Because the power radiated depends on the photon field as well as the electron energy, we need to assume something about the radiation field. If the radiation is isotropic and monoenergetic, at photon energy $\epsilon_o = h\nu_o$, we can call its intensity $I(\epsilon) = F_o \delta(\epsilon - \epsilon_o)$. The angle averaging then gives (after much algebra),

$$P_{ic}(\epsilon_1) = 3\sigma_T F_o F_{ic}(x) \quad (11.16)$$

where we've defined

$$x = \frac{\epsilon_1}{4\gamma^2 \epsilon_o} \quad (11.17)$$

and the kernel function is

$$F_{ic}(x) = x\left(2x \ln x + x + 1 - 2x^2\right) \quad (11.18)$$

Compare the single-particle synchrotron spectrum, from (9.9) and (9.11): it is a narrow function which peaks at $\nu \simeq \nu_c$. Similarly, the function F_{ic} is a narrow function which peaks at $\nu \simeq 2\gamma^2 \nu_o$. Thus we have a "characteristic" frequency for inverse Compton scattering, just as we had for synchrotron.

11.3 Composite spectra

In our previous derivations, we integrated the single-particle spectrum over the distribution of particle energies, to find the volume emissivity (we did this before, for bremsstrahlung and synchrotron). It's more complicated than the previous derivations, because (i) we have to assume an input photon spectrum, and (ii) we really have to worry about whether a photon scatters once, or many times. Instead of doing the general problem, I'll just look at two simple limits.

11.3.1 Nonrelativistic electrons

In this case, each scattering leads to only a very small change in the photon energy: $\epsilon_1' \simeq \epsilon_1\left[1 - (\epsilon'/mc^2)(1 - \cos\theta')\right]$. If we use (11.2), and do angle averaging, we find that the average energy gain per photon, scattering on electrons at temperature T, is

$$\frac{\delta\epsilon}{\epsilon} \simeq \left(\frac{4kT - \epsilon}{mc^2}\right) \quad (11.19)$$

Thus, one scattering can be significant to the radiation source (for instance the microwave background, scattering as it passes through a cluster of galaxies); but it makes little difference to the photon spectrum, as long as $kT \ll mc^2$.

11.3.2 Relativistic electrons, single scattering

The ICS kernel in (11.16, 11.18) peaks at $\epsilon_1 \simeq 2\gamma^2\epsilon_o$ – that comes from the mean scattered photon energy. (It's easy to show that $4\gamma^2\epsilon_o$ is the maximum possible scattered energy; the mean will be somewhat less than this). In most astrophysical applications, the ICS opacity through a source is low, so the photons only scatter once (if at all). Thus IC scattering on electrons at energy γ boosts low-frequency radiation by $\sim \gamma^2$. The low frequency photons might be synchrotron photons from the electrons themselves (in which case this is called "synchrotron self-compton" radiation, SSC); or they might be microwave background photons.

11.3.3 Scattering from power-law electrons

Put a distribution of relativisitic electrons, $n(\gamma)$ again, in a photon field with spectrum $F(\nu')$.[2] If the photons are monoenergetic at ν_o, we have $F(\nu') = F_o\delta(\nu' -$

[2] *Notation alert*: in this section I'm letting ν' be the *incoming* photon energy – so that ν can refer to the scattered photon energy, which is our desired final result.

ν_o) (from above). Looking back to (11.16, 11.18), it will be useful to rewrite the ICS power from a single scattering by a relativistic electron as

$$P(\nu;\nu_o,\gamma) = \frac{3}{4}\frac{\sigma_T}{\gamma^2}F_o\frac{\nu}{\nu_o}f_{ic}(x) \qquad (11.20)$$

where $f_{ic}(x) = 2x\ln x + x + 1 - 2x^2$. If we replace the monoenergetic photon field by a spectrum $F(\nu')$, we can find the emergent (singly) scattered ICS spectrum by integrating over the photon spectrum and the energy distribution:

$$4\pi j_{ic}(\nu) = 3\pi\sigma_T \int\int n(\gamma)\frac{1}{\gamma^2}\frac{\nu}{\nu'}F(\nu')f_{ic}(x)d\gamma d\nu' \qquad (11.21)$$

where $x = \nu/(4\gamma^2\nu')$, as before. Because this is similar to the integrand we encountered in our synchrotron analysis, we can use a similar variable transform. Let x and ν be the integration variables now; for our power-law electron distribution take

$$n(\gamma) = n_o\gamma^{-s}$$

as usual. With this, the integral in (11.21) becomes

$$\begin{aligned} j_{ic}(\nu) =& \frac{3}{4}2^s\sigma_T n_o\nu^{-(s-1)/2}\int F(\nu')(\nu')^{(s-1)/2}d\nu' \\ &\times \int x^{(s-1)/2}f_{ic}(x)dx \end{aligned} \qquad (11.22)$$

This looks horrible, yes; but we're almost there. The last integral, over x, is just a number, because $f_{ic}(x)$ is a simple function. In addition, the first integral, over ν', depends only on the photon spectrum. If we specify $F(\nu')$ (for instance the black body spectrum of the microwave background; or maybe the synchrotron spectrum of the electrons themselves), this first integral can be worked out. Thus, we have the important result: the ICS spectrum from a power-law electron distribution, for single scattering, obeys

$$j_{ic}(\nu) \propto n_o\nu^{-(s-1)/2} \qquad (11.23)$$

That is: if the electrons are a power law, the ICS spectrum is also a power law, with spectral index $(s-1)/2$. This is, of course, the same spectral index as that for synchrotron emission from the same electrons – only we must remember that the ICS photons come out at much higher energies.

References

Once again, this is mostly from my own notes; but you can find more details in

• Rybicki & Lightmann.

Key points

• Compton scattering: what it is, physically, and what "inverse" is.

• Single particle ICS, power *and* spectrum.

• ICS from thermal (nonrelativistic) electrons.

• ICS from relativistic, power-law electrons.