12 Pulsars: overview and some physics

Carroll & Ostlie present the basic picture in some detail; another good reference is Longair's *High Energy Astrophysics*. In these notes I'll be brief about the basics, and emphasize the physics inside the basic model, as well as newer work on high-energy emission and pulsar winds.

12.1 The basic picture

What can cause a star's brightness to pulse as quickly as 100 times a second? That was the immediate question when pulsars were discovered. Stars with longer-period variability are "beating" – that is undergoing body-mode oscillations. But we know a fair bit about normal modes in stars, and none can have so high a frequency. So we must consider rotation; perhaps a hot spot on the star rotates past our line of sight? This was more promising ... but the most compact star then known, a white dwarf, is too big. Remember the rotation rate is limited by lightspeed at the star's surface, as well as by the stability (gravitational binding energy) of the star. Only neutron stars – which were predicted by theory but not yet proved to exist – could explain such a short period.

12.1.1 The cartoon

Thus the basic picture was born: an isolated pulsar is a rapidly rotating neutron star with a small, radio-loud "hot spot". But why is there a "hot spot"? Why does the star's rotation slow down? We think the star is strongly magnetized. We expect this to follow if the NS is made in the core collapse of a supernova; flux freezing will lead to a very strong magnetc field in the NS. But this can, in principle, answer both questions. Two things cause the star to slow down. A rotating magnetic field emits magnetic dipole radiation; by energy conservation this must lead to spindown. If the star sits in vacuum this is the only energy loss. If it sits in the ISM, however, there will also be some torque between the star and the ISM, probably mediated by the magnetic field and/or the plasma outflow from the star (as discussed below).

If the star is strongly magnetized *and if the magnetic dipole is set at an angle to the rotation axis*, we also have a ready cartoon for why it pulses, as follows. Close to the star the magnetic field will be dipolar and will rotate with the star, as will any plasma which is tied to these dipolar field lines. However, at the *light cylinder* radius, that is $r_{LC} = c/\Omega$ if Ω is the rotation rate, the plasma cannot corotate with the star. Magnetic field lines which start close to the magnetic pole will not be able to turn around ("close") before they reach r_{LC}; rather they must connect to the B field of the local ISM. Plasma can therefore flow out along these *open field lines*. If the B field is dipolar, field lines within angle $\sim (R_*/r_{LC})^{1/2}$ of the magnetic axis will be open. If this outflowing plasma can – somehow – make intense radio emission, which is strongly beamed forward along the open field lines, then we will see strong radio pulses only for the fraction of the rotation period when the beamed radiation intersects our line of sight.

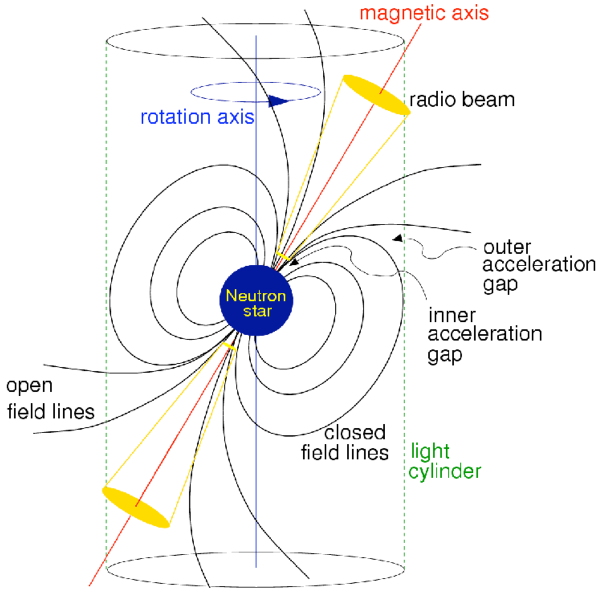

Figure 2.1.1 Cartoon of the standard picture of a pulsar. From Ransom & Condon, NRAO.

12.1.2 And some details

- **Typical numbers for single pulsars.** The star's radius $R_* \sim 10$ km (from NS models). The light cylinder is typically at $\sim 1000 R_*$ (but this of course depends on the rotation rate Ω). The open field line region is a few degrees across at the star's surface. From the duty cycle of the radio pulses, and the assumption that the radio-loud plasma fills the open field line region, we infer the radio emission comes from $\sim 3 - 30 R_*$ above the surface.

- **Energetics**. An isolated pulsar (one not in a binary system) is living off its rotational energy, In terms of its rotation rate Ω and the associated period $P = 2\pi/\Omega$, this is

$$\begin{aligned} E_{rot} =& \frac{1}{2} I\Omega^2 = \frac{2\pi^2 I}{P^2} \ ; \\ \dot{E}_{rot} =& I\Omega\dot{\Omega} = -4\pi^2 I \frac{\dot{P}}{P^3} \end{aligned} \tag{12.1}$$

As you remember (yes?) from basic mechanics, I is the moment of inertia; if the NS were a homogeneous sphere, we would have $I = 2MR^2/5$. Pulsar people generaly take $M \gtrsim 1M_\odot$, and $R \sim 10$ km. These are probably pretty good guesses, although details of the as-yet-unknown equation of state in the star's core can matter here. The 2/5 factor in I is much less certain, due to our ignorance of the internal state of the star and the importance of general relativity in the star's structure. Many authors choose $I \simeq 10^{45}$ (cgs) as a "typical" value, and scale to I_{45}. Collecting these estimates, we find a large range for the power source, $\dot{E}_{rot} \sim 10^{32} - 10^{38}$ erg/s, for old to young pulsars.

• **How strong is the B field?** Remember we have no direct measure; and that very few NS sit in vacuum. Everyone in the field ignores this latter, and assumes that magnetic dipole radiation dominates the spindown. To remind you: magnetic dipole radiation comes from the time-dependence of the star's magnetic moment, $\mathbf{m}$. If this time dependence comes from the star's rotation, we get for a star rotating at Ω and a magnetic axis oriented at α relative to the rotation axis,

$$P_{mag\ dip} = \frac{2|\ddot{\mathbf{m}}|^2}{3c^3} = \frac{2}{3}\frac{(\Omega^2 m)^2}{c^3}\sin^2\alpha \tag{12.2}$$

Now, the magnetic moment is connected to the magnetic field at the star's surface and magnetic pole by $m = B_* R_*^3/2$. But P and $\dot{P}$ can be carefully measured; so, by equating $\dot{E}_{rot}$ (from 12.1) to $P_{mag\ dip}$ (12.2), we can "derive" the B field: $B_* \simeq 3.2 \times 10^{19}(P\dot{P})^{1/2}$. Putting in typical P's and $\dot{P}$'s for single pulsars, we find $B_* \sim 10^{11} - 10^{13}$ G is the expected range. Finally, note that things get interesting when the field is close to the quantum field, defined by $\hbar e B_{cr}/m_e c \sim m_e c^2$: $B_{cr} \sim 4.4 \times 10^{13}$G.

12.2 Spin a magnetic field

If the basic picture – a rapidly rotating, strongly magnetized neutron star – is correct, then some striking physics follows. If you spin a B field, you generate an E field. Think about $\partial\mathbf{B}/\partial t$, let $\hat{\mathbf{z}}$ be the rotation axis,[1] and remember that the rotation velocity is $\mathbf{v}_{rot} = \Omega \times \mathbf{r}$. The E field generated, measured in the inertial ("lab") frame, is

$$\mathbf{E}_{co} = -\mathbf{v}_{rot} \times \mathbf{B}/c = -(\Omega \times \mathbf{r}) \times \mathbf{B}/c \tag{12.3}$$

(I'm calling this the "corotation field", for reasons explained below). The important issue for the local physics is whether this E field is felt, at full strength, by the plasma around the star, and how that plasma responds.

To think about the impact of this on the NS, consider two scenarios. Both are still current in the field, and both lead to strong $\mathbf{E}$ fields, and high particle energies, in the open field line region.

12.2.1 Star in vacuum

Inside the star, we assume the matter and field corotate. The free charge must arrange itself into regions of positive and negative charge, so that the resulting E field just balances the $\mathbf{v} \times \mathbf{B}$ force. Thus, there must exist a *physical* $\mathbf{E}$ field, equal to that in (12.3), so that the net force measured in the corotating frame is zero. But this physical field has a non-zero divergence, so must be supported by a local charge density:

$$\rho_{co} = \frac{1}{4\pi}\nabla \cdot \mathbf{E}_{co} \simeq -\frac{\Omega \cdot \mathbf{B}}{2\pi c} \tag{12.4}$$

(the $\simeq$ means I'm dropping terms $\sim v_{rot}/c$; thus this is valid only well inside the light cylinder). Thus: the matter inside the star must have a net charge density, which is quadropolar: one sign towards the two poles, the opposite sign towards the equator.

Outside of the star, the $\mathbf{E}$ field will be a vacuum solution (by assumption, there are no charges outside), determined by the charge distribution of the star. But we know that from basic E&M, we can just solve Laplace's equation ($\nabla^2\Phi = 0$, if $\mathbf{E} = -\nabla\Phi$) in the vacuum region, *matching the potential and tangential field at the star's surface*. Those solutions can be worked out, but I won't bore you with the details. The interesting part of the solution is strength of the field, $E \sim B\Omega R_*/c$, and the fact that the $\mathbf{E}$ field in the polar cap region has a strong component parallel to $\mathbf{B}$ (call it $\mathbf{E}_\parallel$). From this, we can infer that any free charges in the region will be accelerated to high energies – at least in the open field line region. It may be that these strong fields can even pull charges from the surface of the star (which is mostly but not totally neutrons) ... so that the external vacuum may not last for long.

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[1] I can't do bold Greek; so please pretend Ω is a vector, $\Omega\hat{\mathbf{z}}$.

12.2.2 Filled magnetosphere

Consider the other alternative, that the region above the star's surface is filled with plasma. We know the *charge* density needed if this plasma is to corotate with the star; it's just (12.4). Most authors assume that the plasma within the closed field line region has just this charge density, so that the free charge shields the rotation-induced E field, giving the net $\mathbf{E} = 0$ (measured in the corotating frame). With no net force, the plasma in the region will be static, again relative to the corotating frame. In fact, consider the consequences if the local plasma differed from ρ_{co}: there would be a finite unshielded E field, and the free charges in the plasma would move so as to cancel the field. Such a situation would not be stable, nor likely to last for very long. Thus corotating plasma is likely on closed field lines.

The situation in the open field line region can be more interesting, however. Charges leaving the star's surface must start at low velocity, and go through an acceleration region in which they reach their final energy. If the flow is steady, the charge density must vary inversely with the particle speed. In addition, particles can leave the star along the open field lines, presumably at high speeds. If the particle outflow carries a net charge, a *current* is driven out from the star; as with any electrical system, current density is sensitive to the global circuit (return path, driving voltage and net resistance). From these arguments we suspect that the rotation-induced $\mathbf{E}$ is *not* fully shielded in at least part of the open field line region. Any unshielded $\mathbf{E}_{\|}$ will accelerate particles and drive the outflow/current.[2]

12.3 Radio emission and the pair cascade

Pulsars were discovered by their very intense, pulsed radio emission. Many, many papers have been generated about observations and models of this emission. However, it turns out to be only the small tail of the dog: the total radio power is a very small fraction of the spin-down power, $\dot{E}_{rot}$. In addition, we don't know what causes the radio emission. From the observed very high brightness temperatures ($T_B \sim 10^{35}$ might be typical), we know the emission cannot be due to any of the incoherent processes which apply elsewhere in astrophysics. No plasma can be physically so hot (why?? what would happen to the particles??) so T_B cannot be a physical temperature (as it would if it were thermal emission); nor can this be synchrotron radiation (remember the $T_B \sim 10^{12}$K limit for synchrotron, which we saw earlier). Thus we must be seeing a *collective* or *coherent* emission mechanism; and these are far from understood.

Despite lack of a solid model, a complex scenario has evolved to describe where and how the radio emission is likely to occur. To start, let's stay within the open field line region, where we have just argued that charges are accelerated to very high Lorentz factors. There are two ways in which these particles can emit very energetic photons.

- One way is *curvature radiation*. The charges must follow the magnetic field lines (their gyroradii are tiny; in fact their gyromotion is quantized). The field line curvature makes the particles emit curvature radiation. To understand this, go back to the arguments synchrotron radiation; we can apply them here to curvature emission. The characteristic photon frequency is $\sim 3\gamma^3 c/4\pi\rho_c$, if ρ_c is the radius of curvature of the field lines. The radiated power, integrated over frequency, is $\sim e^2 c\gamma^4/\rho_c^2$ (erg/s per particle). For $\rho_c \sim 10$ km, and the particle energies above, the curvature emission photons come out in the γ-ray region, in particular above the electron rest mass energy.

- Another alternative is *inverse Compton scattering*. The pulsar itself is warm (we know they are thermal X-ray sources). The primary charges can Compton scatter the thermal photons to higher energy, again making γ ray photons above $m_e c^2$. [NOTE: in this situation we cannot simply argue the scattered frequency $\sim \gamma^2 \nu_o$, because of the special geometry, with the photons and charges travelling nearly in the same direction. Doing the calculation carefully does verify that some of the scattered photons are hard enough to be interesting, however.]

Either one of these mechanisms will generate photons which are energetically capable of one-photon pair production, *via* $\gamma + B \rightarrow e^+ + e^- + B$. Recall that this mechanism has a low-B threshold, $h\nu B \gtrsim 0.1 m_e c^2 B_{crit}$, which is very likely satisfied in the high B fields near the pulsar polar cap. Once created, the leptons probably have enough energy to make more energetic photons (through synchrotron ra-

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[2] The real question is what the net, unshielded, potential drop is, and thus particle energies are reached. Typical models suggest Lorentz factors $\gamma \sim 10^6 - 10^7$ are reached; but I wouldn't suggest overmuch confidence in this, the situation is complicated and still not well understood.

diation, most likely, also possibly further curvature and IC emission). Thus a pair cascade occurs ... and is thought to continue until most of the primary beam energy is converted to a dense pair plasma. Models suggest typical Lorentz factors of the pairs $\sim 10^2 - 10^3$.

Now what about the radio emission? As noted above, we need some collective process. The story becomes less clear at this point, as collective plasma emission is not well understood. A general guess is that the pair plasma is a necessary part of the picture. For instance, *plasma turbulence* may be involved; the charges in strong (large-amplitude),turbulent plasma waves can show collective behavior and emit intense radio pulses. If there is a residual $\mathbf{E}_{||}$ in the pair-cascade region, it will generate relative streaming of the electrons and positrons; such streaming is known to lead to plasma turbulence.

12.4 High altitudes and currents

The ideas in the discussion above have been around for quite awhile, about as long as pulsars have been known. In recent years, new telescopes (X and γ ray) have expanded our picture of these stars and their interaction with their immediate environment.

12.4.1 High energy emission

Pulsars are now commonly detected in X-rays and γ-rays. To date, about 30 pulsars have been detected in X-rays and similar numbers in γ-rays (out to 100 MeV). These tend to be the young ones, which will have the largest $\dot{E}_{rot}$, and thus (possibly) have the strongest high-energy emission; thus we might guess that most or all pulsars would show high-energy emission if we had instruments sensitive enough to see them.

The high-energy emission is pulsed, but less narrowly so than is the radio emission. Thus, either the high-energy emission comes from a different location in the star, or (if contiguous with the radio emission) it is less strongly beamed. Many authors suggest this radiation comes from higher altitudes than the radio emission, possibly even from the light cylinder region.

For the stars well-measured up to now, we know that the radio emission is a very small part of the total power; most of the luminosity comes out at high energies, above an MeV. While uncertainties about distance and beaming factor make absolute power estimates difficult, we think that the bolometric power may be comparable to the spindown power. That means that pulsars are quite efficient at converting rotation energy to hard photons; and that the radio emission – the observation which first detected these stars – is but the small tail on a much larger dog.

What type of radiation are we seeing? Normal, incoherent synchrotron (from highly relativistic particles in the star's strong B field) seems to work well for the Xrays. Leptons in the pair cascade are created with finite pitch angles, and lose their energy to synchrotron radiation, much of which comes out as Xrays. The γ-rays are thought to be the leftovers of the pair cascade process, hard photons which escape the star without being turned into pairs.

12.4.2 The pulsar circuit?

Here's an important piece of unanswered physics in pulsar models: what does the current do? That is ... the basic low-altitude model says that free charges are accelerated away from the star's surface. This is what starts the cascade that leads to the radio emission. But this is a current; the rotating star acts as a unipolar dynamo (as we discussed earlier). But the star can't build up a net charge (why?) – so the current must close somehow. How this happens has been the topic of discussion ever since these stars were discovered.

Your author does not claim to know the answer ... but here's a speculation. We know that charges flow most easily along magnetic field lines – and undergo very little electrical resistance along the way. But because the polar current starts out along the open field lines, which must connect to the ISM, we also know the current must move across field lines, in order to connect to other field lines which return to the star. This is likely to happen in the outer magnetosphere (rather like the cross-field charge flow which leads to earth's aurora). Further, cross-field motion is likely to be more resistive; this dissipated energy may come out as observable radiation. Might this be connected to the high-energy pulsed emission, or even to a high-altitude component of the radio emission?

12.5 Winds and nebulae

Now, let's move out to larger scales, past the light cylinder. The standard pulsar model, above, predicts a strong Poynting flux radiated out from the star, and also an outflow of magnetized, relativistic particles. One might expect this energy outflow to couple to normal matter (to mass-load, somehow, when it reaches the ISM); will it drive a wind out from the star? In the inner regions, at least, this will be a relativistic, strongly magnetized outflow, with significant angular momentum – so we should not expect it to be spherical. Rather, it should come out perpendicular to the rotation axis, even if it is initially driven by the plasma outflow from the star's magnetic poles.[3]

12.5.1 Pulsar winds

This idea has been around for quite awhile; thanks to recent technology (mostly X-ray satellites) we are now seeing evidence of these winds. The data are striking. For older pulsars (those not currently within SNR), we sometimes see structures in the nearby ISM which are clearly *bow shocks* associated with the star's high-speed motion through the ISM. We infer that an unseen wind from the pulsar creates the pressure balance that leads to the observed bow shock. From the standoff distance of the bow shock we can estimate the wind energy, and compare it to standard models of the pulsar.

For young pulsars (those still within their SNR), recent CHANDRA images directly reveal the outflow from the pulsars (the Crab and Vela pulsars are the best examples here). These outflows are complex: they show *jets*, which presumably come out along the star's rotation axis (this is the only symmetry axis in the system), and *equatorial winds*, which probably arise from the combined effects of the star's strong magnetic field and its rapid rotation.

12.5.2 Pulsar wind nebulae

The wind coming out from the pulsar is (we think) still highly relativistic, with charged particles moving almost exactly along the magnetic field lines. Thus it should be nearly invisible (how would it radiate), except for its dynamical effects (such as the bow shocks). But what happens when it encounters the local ISM? If that ISM is dense enough, the wind will shock down, and the particles will be "thermalized" (they will gain a significant component of energy transverse to the local magnetic field). Thus, the shocked wind will become visible, as a synchrotron source. This makes what is called a *pulsar wind nebula*, PWN (we've already mentioned these in Chapter 7).

To date there are a several good examples of PWNe; most of them are inside supernova remnants (which provides the high ambient density/pressure that creates the thermalizing shock in the wind). The Crab Nebula is the most striking example of this phenomenon: the shocked pulsar wind fills the nebula, and pushes a shell of cooler matter outwards. This shell is made up of the original SN ejecta and ISM which has been accumulated along the way; it's what we see as the optical and radio nebula.

12.6 Magnetars and Anomalous pulsars

Here's another new area. Based on some recent discoveries, of strong X/γ-ray flares, and/or strongly pulsed X-ray emission, the picture described above is being pushed in two ways. Two characteristics define these unusual objects.

- They have extremely strong magnetic fields, $B \sim 10^{14}$G. This is well above the quantum critical field, $B_{crit} \sim 4\times10^{13}$G – and in the regime where all kinds of interesting physics happens (photon splitting, vacuum birefringence, ...). Such a strong field may be too large to be due to simple flux freezing in the SN collapse of the parent star; various authors discuss dynamos taking place during the SN collapse.
- They are *not* living on their rotational energy: their strong flares have $L \gg \dot{E}_{rot}$. Where, then, does their energy come from? We think it's magnetic – that the very strong B field inside the star can occasionally break through the crust, in a cross between a "starquake" and a very strong "stellar flare". Once this flux tube emerges into the magnetosphere, reconnection will go, releasing the magnetic energy as X-ray and γ-ray radiation.

Because of the way these objects have been found, they are often called SGRs (Soft Gamma Ray Repeaters) or AXPs (Anomalous X-ray Pulsars), and appear to be different objects; but the growing consensus is that these are both observational variants of the magnetar phenomenon. Although none of these objects were initially found in radio, one or two have now been detected to have "normal" radio pulses as well.

Your author doesn't know just how magnetars fit into the general pulsar picture, they are too new – but their

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[3] Picture holding a rigid water hose at some angle, while you spin around your vertical axis. Which way will the water go?

existence points out that the range of extreme conditions which can exist on or around neutron stars seems to be more diverse than we've understood up to now.

Key points

- The basic picture, a pulsar as a rotating magnetized neutron star;
- Our cartoon of the pulsar magnetosphere: what is the plasma doing?
- The pair cascade, how it fills the magnetosphere;
- How the pair-filled magnetosphere might make radio and high-energy emission;
- Pulsar winds and nebulae.
- Magnetars.