3 Some Radiation Basics

In this chapter I'll store some basic tools we need for working with radiation astrophysically. This material comes directly from Rybicki & Lightman ("RL"), where you can find a more complete discussion of it all.

Our apporoach will be ray optics. You remember that radiation can be approached as EM waves, as discrete photons, or in the 'ray optics' limit – we're thinking in terms of ray optics here. Some of this material will look pretty dry to you ... as you go along, look for these important quantities, which will be useful tools for us later.

Intensity, I_ν (equation 3.1)

Flux, F_ν (equation 3.2)

Energy density, u_ν (equation 3.4)

Intensity in thermal equilibrium (TE), B_ν (equation 3.17)

Emission coefficient (per solid angle), j_ν (equation 3.23)

Emissivity, $\epsilon_\nu \rightarrow 4\pi j_\nu$ (equation 3.24)

Absorption coefficient, κ_ν (per solid angle; equation 3.26)

Source function, $S_\nu = j_\nu/\kappa_\nu$ (equation 3.29)

Opacity, τ_ν (equation 3.27)

OK, here goes.

3.1 Radiation: some important definitions

We begin with some important definitions.

• **Intensity**. Consider a little (differential) area $d\mathbf{A} = dA\,\hat{\mathbf{n}}$, with a radiation beam passing through it. At a particular frequency ν, the energy passing through $d\mathbf{A}$ in a particular direction (θ, ϕ) (measured relative to $\hat{\mathbf{n}}$), per frequency range $d\nu$, per time dt, and per solid angle $d\Omega$, is given by

$$dE = I_\nu \, dA \, dt \, d\nu \, d\Omega. \tag{3.1}$$

This relation implicitly defines our basic quantity, the **intensity**: I_ν[1] Most of our other quantities are defined in terms of I_ν. Intensity is also called specific intensity, brightness or surface brightness. In cgs, its units look like erg cm^{-2} Hz^{-1} s^{-1} str^{-1}.

[1]NOTATION ALERT: I_ν is traditional notation in this field, and means "I is a function of ν". So, $I(\nu) \leftrightarrow I_\nu$, and ditto for j_ν, κ_ν, etc.

• **Flux** is the net energy passing through dA, in all directions:

$$F_\nu = \int I_\nu \cos\theta \, d(\cos\theta) \, d\phi \tag{3.2}$$

(with units erg cm^{-2} s^{-1} Hz^{-1} or W m^{-2} Hz^{-1}).

> • **Heads up here:** Flux and intensity are similar but not the same; you'll need to be able to work with both. There is some detailed discussion in Appendix I to this chapter. Basically, intensity is what is shown in an image of a source and flux is what you get if you integrate everything in the image.

• One can also define a **mean intensity**, averaged over all solid angles:

$$J_\nu = \frac{1}{4\pi} \int I_\nu \, d(\cos\theta) \, d\phi \tag{3.3}$$

• The **energy density** is clearly related to the intensity by a factor of lightspeed. The most useful definition is in terms of the angular mean.

$$u_\nu = \frac{4\pi}{c} J_\nu = \frac{1}{c} \int I_\nu \, d(\cos\theta) \, d\phi \tag{3.4}$$

The units are erg cm^{-3} Hz^{-1} (of course).

• **Frequency integrated.** You should also note that each of the quantities above can be integrated over frequency:

$$F = \int F_\nu d\nu \; ; \quad I = \int I_\nu d\nu \; ; \quad u = \int u_\nu d\nu \tag{3.5}$$

and so on.

3.2 Thermal equilibrium: an ideal gas

You've probably seen this elsewhere; I'll just review the basics here. Consider a small system – say, one atom in a gas – which is in *thermal contact* with a large system. That means that energy exchange is allowed, most likely through collisions with the rest of the gas. Let the big system – the so-called "reservoir"– have a temperature T. The fundamental result of classical thermodynamics is that the probability of finding the small (test) system in a state of energy E is

$$\mathcal{P}(E) \propto e^{-E/k_B T} \tag{3.6}$$

This is the *Boltzmann factor*; the proportionality constant is used to normalize the probability, in a specific system (as, below).

Now, let's apply this to an ideal, monatomic gas. Let the test system be a single atom in the gas, and let the rest of the gas be the reservoir, at T. Each atom has a mass, m, and a random velocity, $\mathbf{v}$; the energy associated with this velocity is $E(\mathbf{v}) = \frac{1}{2}mv^2$. (This could of course describe a plasma as well as a neutral, atomic gas). The probability that a particle has energy E is then

$$\mathcal{P}(\mathbf{v}) \propto e^{-mv^2/2k_B T} \tag{3.7}$$

We want to use this to derive the distribution of particle velocities. In addition to the Boltzmann factor, we need to know the number of ways in which a particle of velocity $\mathbf{v}$ can have $E(\mathbf{v})$. If the gas is isotropic – if there are no restrictions on the possible orientation of the velocity vector – then the factor which weights the Boltzmann factor is just the number of possible directions the velocity vector can point. That is, $\mathcal{P}(v) \propto 4\pi v^2 e^{-mv^2/2k_B T}$. Now,if we normalize the answer to the total number density, so that

$$n = \int_{\mathbf{v}} \mathcal{P}(\mathbf{v}) d^3\mathbf{v} = \int_0^\infty f(v) dv \tag{3.8}$$

defines the *distribution function*, $f(v)$, we end up with

$$f(v) = 4\pi n \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_B T} \tag{3.9}$$

which is (one way to write) the *Maxwell-Boltzmann distribution* of particle velocities.

Taking means (or moments) of this distribution, we find the mean particle velocity,

$$\langle v \rangle = \int_0^\infty v f(v) dv = \left(\frac{8k_B T}{\pi m}\right)^{1/2} \tag{3.10}$$

and

$$\langle E \rangle = \int_0^\infty \frac{1}{2} m v^2 f(v) dv = \frac{3}{2} k_B T \tag{3.11}$$

We can extend this type of analysis to find the pressure of the gas. For a simple, MB distribution, this recovers the usual ideal gas law:

$$p = n k_B T \tag{3.12}$$

This analysis can also be used to get the pressure of a relativistic ideal gas, or of a photon gas. I've put the details in Appendix II to this chapter.

3.3 Thermal equilibrium: radiation

Here, we consider a photon gas which is in thermal contact with something at temperature T. That "something" could be the walls of a closed box (the proverbial black body), or a dense gas cloud (which could be a dark interstellar cloud, or a star). As with a particle gas, thermal contact means the photons exchange energy with the "something" through collisions; here, this could mean particle-photon scattering, such as Thompson scattering of photons on electrons; or it could mean that the matter absorbs and re-emits photons. Now, in particle-particle collisions the particle number is conserved (in standard, elastic collisions, anyway). In photon-matter "collisions", on the other hand, the photon number is not conserved; it is quite possible for an atom to absorb one photon and re-emit several, still while conserving energy (for instance, the atom could absorb to the $n = 10$ level and re-emit $10 \rightarrow 8$, $8 \rightarrow 5$ and $5 \rightarrow 1$ photons as it decayed). Thus, in dealing with the particle gas, we normalized the probability function by assuming a certain total number density of particles. For radiation in TE, the number density of photons is *predicted* if we know T.

The analog of the Boltzmann factor for a photon gas is the Planck distribution: the probability of finding a photon at energy $E = h\nu$ is

$$\mathcal{P}(E) \propto \frac{1}{e^{E/k_B T} - 1} \tag{3.13}$$

which, of course, resembles the classical (non-quantum mechanical) Boltzmann factor for energies $E >> k_B T$. We find the density of states allowed at energy E just as we did for particles, by looking at the number of ways photons of wavevector $\mathbf{k} = \mathbf{p}/\hbar = E/\hbar c = 2\pi\nu/c = 2\pi/\lambda$ can be put into a volume. But, recalling standing waves, we remember that a one-dimensional box of length L can contain standing waves of wavenumber $k = 2\pi q/L$ if q is any integer. So, the number of photon states in three dimensions in $(\mathbf{k}, \mathbf{k} + d\mathbf{k})$ is

$$d^3\mathbf{q} = \frac{L^3}{8\pi^3} 4\pi k^2 dk \tag{3.14}$$

From this, we find the density of states per volume by dividing by L^3 and adding the usual factor 2 for the two spin (polarization) states, and express things in terms of ν rather than k:

$$\text{density of states} = 8\pi \frac{\nu^2}{c^3} d\nu \tag{3.15}$$

This is the factor which weights $\mathcal{P}(E)$ (from equation 3.6) to find the density of photons at energy $E = h\nu$. (Note that this is 4π times larger than RL's expression for ρ_s; they are working in density of states per solid angle.) However, it is common to multiply this density by $h\nu$ to find the energy density of radiation in TE at T:

$$u_{rad}(\nu, T) d\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/k_B T} - 1} d\nu \tag{3.16}$$

(In dealing with black body radiation, watch out for units; different authors do different things. u_{rad} in equation (3.16) has units energy/volume-Hz; some authors divide by 4π to get energy/steradian-volume-Hz.)

Another way to express this result is in terms of the energy in radiation crossing a unit area per Hz per time (and usually per steradian). As we saw above, this quantity is the *intensity*, and is related to the energy density in radiation by $I(\nu) = c u_{rad}(\nu)/4\pi$. For (and only for) the specific case of Black Body radiation, the intensity is denoted $B(\nu, T)$ or $B_\nu(T)$, rather than I_ν. For black body radiation, we therefore have

$$B(\nu, T) = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1} \tag{3.17}$$

with units, energy/area-time-steradian-Hz.

Equations (3.16) and (3.17) are the basic result describing radiation in TE. However, several extensions and approximations are standard. First, we can integrate them over frequency to find the total energy (per volume, or per area per time):

$$\begin{aligned} u_{rad}(T) &= \int_0^\infty u_{rad}(\nu, T) d\nu \\ &= \frac{4\pi}{c} \int B_\nu(T) d\nu = aT^4 \end{aligned} \tag{3.18}$$

where the constant $a = 8\pi^5 k_B^4 / 15 c^3 h^3 = 7.56 \times 10^{-15}$ erg/cm^{3}deg^4. In addition, we have

$$B(T) = \int_0^\infty B_\nu(T) d\nu = \frac{ac}{4\pi} T^4 \tag{3.19}$$

and, finally, the emergent flux obeys $F = \pi B(T)$, so that

$$F(T) = \int F_\nu d\nu = \pi \int B_\nu d\nu = \sigma_{SB} T^4 \tag{3.20}$$

where $\sigma_{SB} = ac/4 = 5.67 \times 10^{-5}$erg/cm^{2}deg^{4}s^{-1}.

Finally, a couple of limits of $B_\nu(T)$ are worth noting. For high frequencies with $h\nu \gg k_B T$, we have the Wien limit:

$$B_\nu(T) \simeq \frac{2h\nu^3}{c^2} e^{-h\nu/k_B T} \tag{3.21}$$

which recovers the exponential form. For low frequencies $h\nu \ll k_B T$, we have the Rayleigh-Jeans limit:

$$B_\nu(T) \simeq \frac{2\nu^2}{c^2} k_B T = \frac{2}{\lambda^2} k_B T \tag{3.22}$$

which is very frequently used as an approximation to the black body function in the radio part of the spectrum.

3.4 Radiative transfer

Start with a beam of radiation, described as usual by intensity I_ν. Consider such a beam, from some background source, hitting a slab of material. It will be absorbed by the material as it passes, and the material may well also emit radiation into the beam.

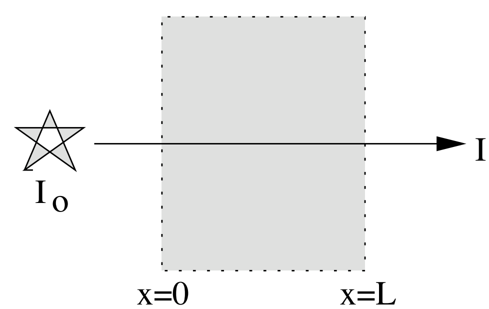

Figure 3.1 Geometry of radiation transfer through a slab of material, possibly with a background source I_o.

3.4.1 More definitions

We need more definitions now. In addition to the intensity, I_ν, we need to think about the plasma through which the beam passes.

• The plasma has an **emission coefficient** j_ν, defined in terms of the contribution to a radiation beam (I_ν as it propagates a distance dx:

$$dI_\nu = j_\nu dx \tag{3.23}$$

and the units of j_ν are erg cm^{-3} s^{-1} Hz^{-1} str^{-1}. This is the fundamental radiated power, at frequency ν, from the matter; its details depend on the local physics. Note also that the mean intenstiy J_ν that we saw earlier is not the same as the emission coefficient j_ν.

• For some problems it's more useful to work with the **volume emissivity**,

$$\epsilon_\nu = 4\pi j_\nu \tag{3.24}$$

(This assumes isotropic emission). We'll run into this later.

• We also need the **absorption coefficient** κ_ν,[2] which describes the fractional absorption or scattering of a radiation beam, per unit length dx:

$$dI_\nu = -\kappa_\nu I_\nu dx \tag{3.25}$$

This has units cm^{-1}. It can also be written microscopically (to reveal the physics), in terms of the number density of absorbers n and their absorption cross section σ_ν: $\kappa_\nu = n\sigma_\nu$. Question for the reader: How, then, is the absorption coefficient related to the mean free path of a photon?

3.4.2 Transfer analysis

With these definitions, the basic transfer equation can be written down,

$$\frac{dI_\nu}{dx} = j_\nu - \kappa_\nu I_\nu \tag{3.26}$$

Before solving this, we introduce an important and useful quantity, the *optical depth*:

$$\tau_\nu = \int_o^L \kappa_\nu dx \tag{3.27}$$

where the integral is taken from back to front through the slab of matter. From the discussion of κ_ν, above, we see that $\tau_\nu = L/\lambda$ measures the number of absorption mean free paths through the source. We would expect, then, that a system with $\tau_\nu \ll 1$ would have little effect on any source behind it (that is, it would be nearly transparent), and a system with $\tau_\nu \gg 1$ would be nearly opaque, absorbing most of the light. We can rewrite (3.26) with τ as the independent variable:

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu \tag{3.28}$$

where we have defined the *source function*,

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \tag{3.29}$$

[2]NOTATION ALERT: about half the literature uses α_ν for the absorption coefficient; the other half uses κ_ν, as I do here.

Now, solve (3.28). If we put a source of intensity I_o behind the slab, the formal solution (remember integrating factors?) is

$$I_\nu(\tau_\nu) = I_o e^{-\tau_\nu} + \int_0^{\tau_\nu} S(\tau')e^{-(\tau_\nu - \tau')}d\tau' \tag{3.30}$$

Look at this: the first term is simply the attenuation of the background source by the slab. The variable τ' is a distance through the slab, but it's measured in dimensionless optical depth units. The second term describes the emission of radiation from within the slab, at position τ', and the attenuation of this radiation by the smaller optical depth, $\tau_\nu - \tau'$, between the emission point and the front of the slab.

3.4.3 Optically thick and thin limits

In the important case of a homogeneous source, with $I_o = 0$, (3.30) simplifies to

$$I_\nu(\tau_\nu) = S_\nu\left(1 - e^{-\tau_\nu}\right) \tag{3.31}$$

describing emission only from the cloud/slab itself. This has two important limits:

• *Optically thin*, $\tau \ll 1$: we see

$$I_\nu \simeq S_\nu \tau_\nu = j_\nu L \tag{3.32}$$

This limit just integrates the emissivity through the cloud, without modifying it by internal absorption.

• *Optically thick*, $\tau \gg 1$:

$$I_\nu \simeq S_\nu \tag{3.33}$$

In this limit, the emergent intensity is just equal to the source function. NOTE this may be quite different from the internal emissivity; transfer through the source has modified the spectrum.

From the optically thick limit we can make a couple of important connections.

• Consider the case when radiation is in TE with the local plasma. This means $I_\nu \to B_\nu$ (the Planck function, (3.17 or its limits); and also $\tau_\nu \gg 1$ (because you need lots of collisions, *it i.e.* optically thick, to gain TE). Thus, from (3.31), we expect $S_\nu \simeq B_\nu$ (the source function approaches the Planck function), and from this we derive an important relation:

$$j_\nu \simeq B_\nu \kappa_\nu \tag{3.34}$$

This is called **Kirchoff's law**. It says: in the optically thick limit, *if we can also assume thermal equilibrium*, the emissivity and absorption are related through the Planck funciton.

• Astronomers working at radio frequencies commonly quote the intensity, I_ν, in terms of the **Brightness Temperature**, T_B, defined by

$$I_\nu = 2\frac{\nu^2}{c^2} k_B T_B \tag{3.35}$$

But from comparing (3.22) and (3.35), we find that $T_B \to T$ as the source becomes optically thick: the brightness temperature approaches the physical temperature. (Question for you: if the source is optically thin, how are T and T_B related? How does your answer depend on the conditions in the source?)

3.5 Appendix I: some examples with intensity

Working with intensity, flux, etc, can be confusing (at least to your author!); so I'm putting some specific examples here – mostly directly from RL.

3.5.1 Isotropic radiation field

If the radiation field is isotropic life is simple: $J_\nu = I_\nu$ (is that obvious?) and $F_\nu = 0$ (there's no net energy flow; there's as much going "out" as going "in"). Also, by inspection of (3.4), we see that $u_\nu = 4\pi I_\nu / c$.

3.5.2 Intensity is constant along a ray

We should note one important fact: in the absence of absorption or emission, the intensity I_ν is constant along any ray. RL present one (rather formal) derivation of this, illustrated in Fig 3.2, which I summarize here. The key points are that intensity is defined per solid angle, and that energy is conserved. Think about the set of rays passing through both dA_1 and dA_2. The energy in that set of rays is

$$dE = I_1\, dA_1\, dt\, d\Omega_1\, d\nu = I_2\, dA_2\, dt\, d\Omega_2\, d\nu \tag{3.36}$$

But, thanks to the inverse square law, $d\Omega_1 = dA_2/R^2$, and $d\Omega_2 = dA_1/R^2$. Thus, because the same dE passes through both little areas, we must have $I_1 = I_2$. Q.E.D.

Wait .. does this seem unphysical? What about the inverse-square law that we know applies to radiated power? The key is that intensity is not the same as flux – and flux satisfies the $1/R^2$ law. This is contained in (3.36), because the solid angle $d\Omega = dA/R^2$; so that the energy *per area* passing through any dA falls off $\propto 1/R^2$.

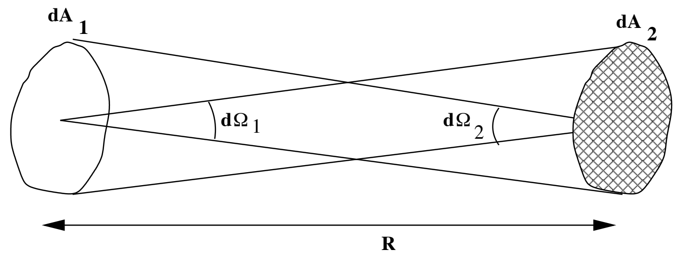

Figure 3.2 One way to establish the constancy of I along a ray. Two little areas, dA_1 and dA_2, are separated by R. dA_1 subtends a solid angle $d\Omega_2$, as seen by 2; and vice versa for dA_2 as seen by 1. Following RL Fig 1.5.

Or, if this argument isn't very transparent, you might prefer the next example.

3.5.3 Flux from a sphere

Here's a nice example, to verify that radiative flux does indeed obey the inverse square law. Put yourself at distance D from a sphere of uniform brightness B; that means all rays leaving the surface have the same intensity, $I = I_o$, independent of direction. (The geometry is shown in Figure 3.3).

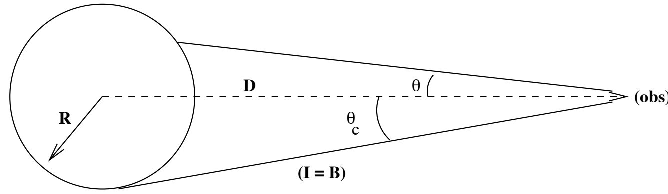

Figure 3.3 The geometry used for calculating the flux from a uniformly bright sphere. The observer is at a distance D away; the sphere radius is R; the radius subtends an angle θ_c as seen by the observer; the intensity is assumed uniform, $I = I_o$, along every ray that leaves the surface. Following RL Fig 1.6.

The intensity you see, then, is $I = I_o$ from rays which intersect the sphere; and $I = 0$ from other angles. The flux you observe is thus

$$F = \int_0^{2\pi} d\phi \int_0^{\theta_c} I(\theta,\phi) \cos\theta d\cos\theta \tag{3.37}$$

But this is easy to integrate:[3]

$$F = \pi I_o \left(1 - \cos^2\theta_c\right) = \pi I_o \sin^2\theta_c \tag{3.38}$$

[3] Just to be more difficult ... some authors (*e.g. Mihalas* "absorb the π (in 3.39) into the definition of flux", and call it "astrophysical flux". I think the moral is, be careful when you go from author to author – JAE.

This nicely recovers the inverse square law as long as the distances involved are not cosmological:

$$F = \pi I_o \frac{R^2}{D^2} \tag{3.39}$$

Also, note that at $R = D$ when you're right at the surface of the star), the flux is

$$F(D = R) = \pi I_o \tag{3.40}$$

We can also invert this solution. Say we observe flux F at earth.[4] The intensity at the source is, clearly,

$$I_\nu = \frac{F_\nu}{\pi}\frac{D^2}{R^2} \tag{3.41}$$

But also, remember $\theta_c = \sin^{-1}(R/D) \simeq R/D$ (this last for a distance source); so the solid angle subtended by a distant source is $\Omega_c = \pi\theta_c^2$. Thus, we can go from the flux (at earth) to the intensity (at the source) by

$$I_\nu = \frac{F_\nu}{\Omega_c} \tag{3.42}$$

3.6 Appendix II: more on pressure

3.6.1 Ideal gases: the pressure integral

We can also use the MB distribution to find the pressure of an ideal gas. (We will derive this is a fairly formal way, to use later on). (For this subsection, we use p for the single-particle momentum, and P for the pressure). The pressure is, of course, the force exerted per unit area from collisions of the gas particles. Consider some "test surface" within the gas, and we can find this force. One particle, with momentum $\mathbf{p}$, approaches this surface at angle θ. When it recoils from the surface, it transfers momentum $\Delta p = 2p\cos\theta$ to the surface. Now, the rate of particles approaching at this $\mathbf{p}$ and this θ is

$$j(p, \theta) = v(p)\cos\theta \frac{1}{2} f(p)\sin\theta$$

(that is, the normal velocity times the number of particles "at θ"; and noting that only half of the particles "at θ" are approaching the surface). The pressure, then, is this rate times the Δp per collision, integrated over all angles and all velocities:

$$P = \int_0^{\pi/2} d\theta \int_0^\infty dp 2p\cos\theta j(p, \theta) \tag{3.43}$$

[4]NOTATION ALERT: the flux at earth is often called S or S_ν, at least in radio astronomy.

If we now assume the gas is isotropic, we can do the θ integral right away, and we end up with

$$P = \frac{1}{3}\int_0^\infty pv(p)f(p)dp \tag{3.44}$$

If we put in the MB distribution, from (3.9), and assume a subrelativistic gas, so that $v(p) = p/2m$, we find $P \propto \int_0^\infty p^2 e^{-p^2/2mk_BT}dp$, and end up with

$$P = nk_BT \tag{3.45}$$

as we expect.

Another interesting limit is that of a relativistic ideal gas, in which the single particle energy $E \gg mc^2$. In this limit, we have $p \simeq E/c$ and $v \simeq c$, so that

$$P \simeq \frac{1}{3}\int Ef(E)dE \tag{3.46}$$

(where we have used $f(E)dE = f(p)dp$). But the integral is just the energy density of the gas, u; so we have

$$P = \frac{1}{3}u \tag{3.47}$$

which is a general result for an internally relativistic gas (whether of particles or of photons).

3.6.2 Radiation pressure

Although not limited to radiation in TE, this is as good a place as any to put the basic facts about radiation pressure. For a general radiation field, we can use the general equation (3.43), with the photon flux written as

$$j(\nu, \theta) = \frac{c}{2h\nu}\cos\theta u_{rad}(\nu, \theta)\sin\theta \tag{3.48}$$

To evaulate P_{rad}, we would have to know the angular distribution of u_{rad} (for instance, the basic radiation-pressure applications, such as an astronaut with a flashlight, or a spaceship with a light sail near the sun, assume a highly directed u_{rad}).

One simple limit is the case of an isotropic radiation field (which is a good description of a radiation field in which the photons scatter, such as the interior of a star – that is, a photon field which is probably also in TE). Here, we can start with equation (3.44), and we can certainly use the relativistic limit; thus, we get

$$P_{rad} = \frac{1}{3}u_{rad} \tag{3.49}$$

in general, for an isotropic radiation field.

The other simple limit is the case of a unidirectional radiation field – for instance radiation from the sun as seen at the distance of the earth. We could more properly talk about *radiation force* here: dimensionally that's the (radiation power)$\times$(surface area of the absorber)/c. One application of this is the luminosity at which the radiation pressure (or force) from some object of mass M balances its gravity. This is the *Eddington luminosity*. If we're talking about ionized gas for which Thomson scattering (cross section σ_T) dominates, the Eddington luminosity is $L_{edd} = 4\pi c G M m_p/\sigma_T$ (which you remember from last term).

References

The discussion about ideal gas laws, pressure integrals, etc, can be found in any basic statistical mechanics book – two good ones are

• Reif, *Statistical and Thermal Physics*; Kittel, *Thermal Physics*.

The material on radiation comes straight from one of the fundamental references in the field,

• Rybicki & Lightman, *Radiation Processes in Astrophysics*

but also

• Mihalas, *Stellar Atmospheres*, has a good discussion of the intensity basics.

Key points

• Basic definitions, I_ν; F_ν; u_ν, and what they mean;

• Radiation in TE: Black body physics

• Radiative transfer: j_ν, κ_ν, and τ_ν: what they are, what they mean.

• Radiative transfer: solutions to $I_\nu(\tau_\nu)$, optically thick and thin limits.

• Brightness temperature and the approach to TE (as τ_ν gets big).