5 Thermal state of the ISM

What determines the energy balance of diffuse astrophysical gases? In general, systems which have had time to reach a thermal balance will have a temperature determined by the balance of heating and cooling rates. For most phases of the ISM (although probably not including the hot, coronal gas), the cooling is due to radiation, from an optically thin gas. The specific radiation mechanisms which cool a cloud or nebula will depend on the composition, the internal excitation states and the temperature of the cloud. This part of the problem is fairly well understood by now, at least for the low-density conditions which describe the ISM. The heating mechanism must also be specified; in general, the ISM can be heated by the local radiation field, and by the local cosmic ray population. Since the rate of energy transfer from either of these mechanisms can depend on the local ionized fraction, we must also consider ionization balance. In this chapter, we will first look at some general features of cooling and heating in this context. We will then look at two well-understood examples, one being the thermal state of an HII region, and the other being the multiphase ISM.

> *NOTE to the student*: This chapter contains quite a few details and a lot of unfriendly-looking equations. That's because we need these details in order to understand the various processes whose balance determines the thermal state of the ISM. In addition to the Key Points at the end of the chapter, I'm adding "Look ahead" comments to each of the major sections .

5.1 Heating and cooling: general considerations

The energetics of astrophysical fluids or plasmas can be more complicated than lab fluids, because *direct* heating and cooling can be important. By "direct" I mean that a small volume element, deep inside a cloud, can exchange energy directly with the outside world via photons or other particles.

> *LOOKING AHEAD*: The biggest result here is the general cooling curve, $\Lambda(T)$, shown in Figure 5.1, and described in the text. The other important bit is the list of processes which contribute to heating the ISM.

5.1.1 Cooling

For energy loss, the most important astrophysical case is radiation. If the plasma or cloud is optically thin, radiation generated inside the volume element escapes the cloud without further interaction. The net energy loss (per volume per time, frequency-integrated) is often called

$$\Lambda(n, T) = n^2 \mathcal{L}(T) \qquad (5.1)$$

and has dimensions, erg $\mathrm{cm}^{-3}\ \mathrm{s}^{-1}$. Note the separation into n^2 and a function $\mathcal{L}(T)$ of T only. It reflects the fact that radiative cooling processes are all two-body processes (electron-ion scattering, electron-atom collisions, etc). The total cooling rate from a thermal astrophysical plasma is due to the sum of all possible spectral lines emitted and also continuous emission due to electron-ion collisions. You've already seen the latter; it's *bremsstrahlung*. The former can be quite complicated – one has to know the excitation state of each level of each atom (which depends on the temperature of the gas and the microphysics of excitation and de-exciation processes) – and must be calculated numerically.

Figure 5.1 shows the result, from a calculation (due to Raymond, Cox & Smith 1977), which adds up all of the different cooling mechanisms, to get what's called the standard *cooling curve*. The main coolants, in the various temperature regions, are:

• $T < 10^4$ K: collisional excitation of fine structure levels of various heavy elements; lines mostly in the IR.

• $T \sim 10^4$ K: excitation of hydrogen lines; the great abundance of hydrogen makes this the dominant coolant in the temperature range where H is significantly excited at or above the first excited state.

• $T \sim 10^4 - 10^7$ K: excitation of optical/UV lines of heavy elements, often forbidden lines.

• $T \gtrsim 10^7$ K: no species are left un-ionized, so bremsstrahlung is the only coolant.

At temperatures $T \gtrsim 10^9$ K, other reactions – such as pair production – become important; very few astrophysical plasmas are this hot, and if they are, other physics is probably important as well.

5.1.2 Heating

Direct heating of astrophysical plasmas isn't so simple, because it depends on the environment of the cloud,

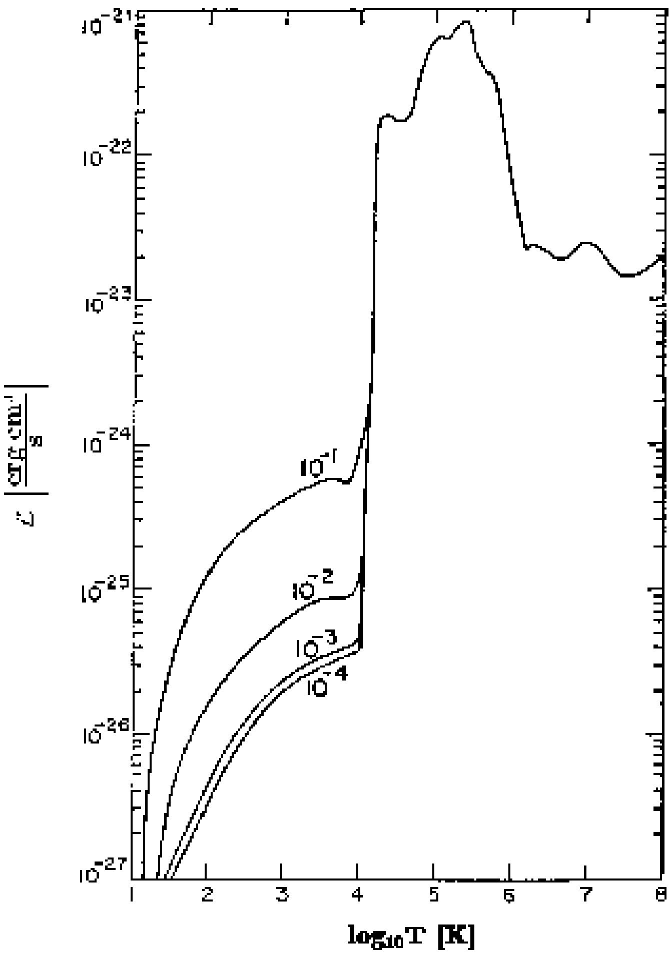

Figure 5.1 Cooling rates as a function of tempoerature, for diffuse astrophysical plasma with solar abundances. At low temperatures the cooling rate depends on the ionization parameter $x \equiv n_e/n_H$, which labels the curves. Note the curve is presented for $\mathcal{L} = \Lambda/n^2$ because radiative processes are two-body (so go as n^2). From Spitzer Figure 6.2.

not just its internal state (density and temperature). We know, in general, the heating sources for the diffuse ISM. Stars are the fundamental energy sources; they lose energy, and supply power to the ISM, by starlight and also through dynamic input from such events as supernovae and stellar winds. What we need to quantify here, however, is the microphysics: how does this energy couple to some small test volume deep within an interstellar cloud? This area is still under discussion. Here I lean on the discussion from McKee (1995), which focuses on the diffuse ISM; but add photoionization heating which is important for HII regions.

• **Cosmic Rays**. These high-energy particles penetrate the cold ISM, and can transfer energy both through ionization of the neutral component and through Coulomb interactions with the ionized component. This is the heating mechanism first suggested by Field, Goldsmith, &Habing (1969), in their original model of the multiphase ISM. Since then other authors have argued that cosmic rays can't supply enough power, and that other possibilities are more important.

• **X-rays**. These have also been suggested, as they can also penetrate the ISM. They heat by ionization. However, most authors seem to agree that their heating rate is also too low to be useful.

• **Magnetic dissipation**. This is attractive but harder to quantify. The idea is that the large-scale galactic field will dissipate locally, either by local reconnection events (similar to solar flares), or by dissipation of MHD waves. The latter in particular could be a heating mechanism for the cool ISM, as the waves could propagate fairly far from their sources before dissipating. I have not seen this treated quantitatively, however, in any ISM modelling.

• **Photoelectric heating**. This seems to be the current favorite. Starlight is abundant in the galaxy (it also has a density $\sim$ 1 eV/cm^3); can it couple to the ISM? Most starlight is too cool to ionize the ISM. It can, however, be absorbed by interstellar grains which then eject electrons, in a standard photoelectric effect. This can be quantitatively effective if very small grains and also large organic molecules (polycyclic aromatic hydrocarbons, PAH's) are included in the models.

• **Photoionization heating**. This will be treated in more detail immediately below. It is unlikely to be important in the diffuse ISM, because there are not very many "loose photons" with $h\nu > 13.6$ eV; but it is the dominant heating mechanism close to hot, young stars (and probably close to active galactic nuclei).

For our purposes here, the total heating rate – whatever it may be – will be called

$$\Gamma = n\mathcal{H} \tag{5.2}$$

and has units erg cm^{-3} s^{-1}. Once again, we've extracted the density dependence as most of the heating mechanisms described here depend on the first power of the density (*i.e.*, on how many atoms are around to be heated).

5.2 HII regions

Let's start with a spherical chicken. That is, we will start with the ionization and thermal structure of a simple, one-star HII[1] region. Consider one hot star (O or B type, with a significant part of its luminosity above the ionization edge of hydrogen, 13.6 eV), sitting in the ISM. The UV photons from this star will ionize and heat the local hydrogen, producing an HII region around the star. Within an HII region, the central star (or stars) will dominate the energy and ionization balances, which simplifies the problem (compared to the more general case of the diffuse ISM). To start, then, we will therefore consider the equilibrium of a purely photonionized nebula.

> *LOOKING AHEAD*: The important features in here are (i) the definitions of ionization and recombination rates (say equations 5.4 and 5.5); their use in ionization balance (equations 5.7, 5.8); the Stromgren sphere (5.17); the general discussion of heating and cooling (§5.2.2); and the final result, 8000-10,000 K "always".

5.2.1 Ionization structure

Let the central star have a spectrum $L(\nu)$ (in erg/s-Hz). At a distance r from the star, the mean intensity is

$$J(\nu, r) = \frac{L(\nu)}{4\pi r^2} e^{-\tau(\nu,r)}. \tag{5.3}$$

Look back to chapter 3: $J(\nu, r) = \frac{1}{4\pi}\int I(\nu, r) d\Omega$ is the mean intensity averaged over solid angle. The expression in (5.3) describes the intensity from the central star, attenuated by some optical depth $\tau(\nu, r)$ to be evaluated below. Photons above $h\nu_1 = 13.6$eV will ionize hydrogen; the ionization cross section is $\sigma_{ion}(\nu) \simeq 6.6 \times 10^{-18} (\nu/\nu_1)^{-3}$ cm^{-2}. We can therefore write down the ionization rate per hydrogen atom,

$$\varphi_{uv}(r) = \int_{\nu_1}^{\infty} 4\pi \frac{J(\nu, r)}{h\nu} \sigma_{ion}(\nu)\, d\nu \tag{5.4}$$

Note that φ_{uv} has units s^{-1}; the ionization rate per volume is $n_{HI}\varphi_{uv}$, units cm^{-3} s^{-1}.

We also need the recombination rate to level j, from the continuum; call it $n_e\alpha_j(T)$ recombinations per second per atom. From the recombination cross section, again assuming a Maxwellian distribution of free electron velocities, $\alpha_j(T)$ is

$$\begin{aligned} n_e\alpha_j(T) &= \int v\sigma_{rec,j} f(v) dv \\ &\simeq \frac{2.1 \times 10^{-11} g_j n_e}{\left[1 + j^2 k_B T/13.6\text{eV}\right)] j^2 T^{1.2}} \end{aligned} \tag{5.5}$$

where g_j is the statistical weight of that level. This can be summed over all levels to get the total recombination rate per atom,

$$\alpha(T) = \sum_j \alpha_j(T) \simeq 2.1 \times 10^{-11} \frac{\Phi(T)}{T^{1/2}} \tag{5.6}$$

where $\Phi(T)$ is yet another order-unity factor, which varies only slowly with temperature in the range $10^2 - 10^5$K.

In equilibrium, the ionization rate will be balanced by the recombination rate, $n_{HII} n_e \alpha(T)$ (recombinations per volume per second). The balance is then given by

$$n_{HI}\varphi_{uv} = n_{HII} n_e \alpha(T) \tag{5.7}$$

Now, the total hydrogen density is $n = n_{HI} + n_{HII}$; if the nebula is mostly hydrogen, $n_e \simeq n_{HII}$. Thus, we can define $x = n_{HII}/n$ as the ionized fraction (so that $n(1-x)$ is the neutral fraction), and write (5.7) as

$$\frac{1-x}{x^2} = \frac{n\alpha(T)}{\varphi_{uv}} \tag{5.8}$$

We note, φ_{uv} is a function of position in this case, from (5.4).

Consider the solutions to (5.8) for x. We know $0 \le x \le 1$, by definition. Now, close to the star, the right hand side of (5.8) will be $\ll 1$. We therefore know $x \simeq 1$, and the quadratic solution of (5.8) will be

$$1 - x \simeq \frac{n\alpha(T)}{\varphi_{uv}} \ll 1 \tag{5.9}$$

Far from the star, the right hand side will be $\gg 1$ (as φ_{uv} drops), and the solution will be

$$x \simeq \left(\frac{\varphi_{uv}}{n\alpha(T)}\right)^{1/2} \ll 1 \tag{5.10}$$

In addition, the region (of r) over which $n\alpha(T)/\varphi_{uv}$ changes from large to small turns out to be small (you can show that this region has width $\Delta r \sim$ the mean free path of a photon in the neutral gas). Thus, we

[1] Notation: HII (called "H-two") refers to ionized hydrogen; HI ("H-one") is neutral hydrogen. And just to be confusing, H_2 (also called "H-two") is molecular hydrogen.

expect the ionization structure to go from fully ionized, to hardly ionized, over a very short distance.

NOW, We want to study the structure of the nebula in a bit more detail, and to find the location of the ionized/neutral transition. To start, we need the optical depth, from the star out to radius r:

$$\begin{aligned}\tau(\nu, r) &= \int_0^r n(1-x)\sigma_{ion}(\nu)dr \\ &\simeq n(1-x)\sigma_{ion}(\nu)r\end{aligned} \tag{5.11}$$

where the last step assumes a uniform density and that $(1-x) \simeq 0 \simeq$ constant (which is true within the ionized region). Now, consider the local radiation density, $u(\nu, r) = J(\nu, r)/c$. Using (5.3), we can write

$$\begin{aligned}&\frac{d}{dr}\left(4\pi r^2 cu(\nu, r)\right) \\ &\quad= \frac{d}{dr}\left(L(\nu)e^{-\tau(\nu,r)}\right) \\ &\quad= -L(\nu)e^{-\tau(\nu,r)}\frac{d\tau(\nu, r)}{dr}\end{aligned} \tag{5.12}$$

And, using (5.11), this becomes

$$\begin{aligned}&\frac{d}{dr}\left(4\pi r^2 cu(\nu, r)\right) \\ &\quad= -L(\nu)e^{-\tau(\nu,r)}n(1-x)\sigma_{ion}(\nu) \; .\end{aligned} \tag{5.13}$$

Now, we can (a) use (5.7) to eliminate $n(1-x)$ on the right hand side; (b) multiply both sides by $d\nu/n\nu$, and (c) integrate over ν:

$$\begin{aligned}&\frac{d}{dr}\left[4\pi r^2 c\int_{\nu_1}^{\infty}\frac{u(\nu, r)}{n\nu}d\nu\right] \\ &\quad= -4\pi r^2 c\frac{\alpha(T)n^2x^2}{\varphi_{uv}}\int_{\nu_1}^{\infty}\frac{u(\nu, r)\sigma_{ion}(\nu)}{h\nu}d\nu\end{aligned} \tag{5.14}$$

At this point, we can identify terms. The quantity in brackets on the left of the equation is the number of UV photons per second crossing the surface at r, $S_{uv}(r)$. The integral on the right side is just $\varphi_{uv}(r)/c$. Thus, the equation simplifies quite nicely to

$$\frac{dS_{uv}(r)}{dr} = -4\pi r^2\alpha(T)n^2x^2 \tag{5.15}$$

For a constant density region, in which $x \simeq 1$, we can easily solve this equation:

$$S_{uv}(r) = S_{uv}(0) - \frac{4\pi}{3}r^3\alpha(T)n^2x^2 \tag{5.16}$$

where $S_{uv}(0)$ is the photon flux at the star.

From this, we can define the *Stromgren radius* as the distance at which the photon flux goes to zero:

$$S_{uv}(0) = \frac{4\pi}{3}R_s^3\alpha(T)n^2x^2 \tag{5.17}$$

This limit defines the Stromgren sphere. We note that – due to the assumption of local ionization balance – the expression for R_s only depends on the *total* number of UV photons, not on their energy, or how many central stars there are, or the ionization cross section, or any number of physical parameters one might have thought interesting. R_s is determined only by $S_{uv}(0)/n^2$. (Remember that $x \simeq 1$ inside R_s). The physical picture is, simply, that the size of the HII region – R_s – is set by the volume inside of which the number of recombinations per second exactly balanced the number of ionizing photons put out by the star per second.

5.2.2 Energy balance and temperature

Photoionization also provides the heating for the nebula. A photon of energy $\nu > \nu_1$ ionizes an atom, and the leftover energy $h(\nu - \nu_1)$ goes to kinetic energy of the free electron. This electron can then share the energy with the ions, through Coulomb collisions, so that this becomes a general heating mechanism for the gas.

The heating rate per HI atom can be written

$$\begin{aligned}&\int_{\nu_1}^{\infty} 4\pi\frac{J(\nu)}{h\nu}\sigma_{ion}(\nu)h(\nu-\nu_1)d\nu \\ &\quad\simeq \varphi_{uv}\langle h(\nu-\nu_1)\rangle\end{aligned} \tag{5.18}$$

where the second expression defines the mean energy transfer per ionization (with the mean taken over the input photon spectrum). Note, we have suppressed the r dependence in $J(\nu)$, to ease the notation. Thus, the heating rate per volume is

$$\Gamma_{uv} = n_{HI}\varphi_{uv}\langle h(\nu-\nu_1)\rangle \tag{5.19}$$

We can again assume ionization balance, and this can be written inc terms of the local density and temperature, as

$$\Gamma_{uv} = n^2x^2\alpha(T)\langle h(\nu-\nu_1)\rangle \tag{5.20}$$

To determine the equilibrium temperature, we want to balance Γ_{uv} against all of the important radiative cooling mechanisms (assuming the HII region is optically thin, which is a good assumption for visible ones – the ones in the pretty pictures). It turns out that there are two important types of coolants – recombination lines from hydrogen (and helium, which is a small correction), and collisionally excited lines from heavy elements. To do the calculation, we have to to compute the cooling rates numerically for these lines (or find someone who has done it already!).

The hydrogen recombination cooling rate is

$$\Lambda_{rec} = n_{HII} \sum_j \int \sigma_{rec,j}(v) v f(v) \frac{1}{2} m_e v^2 dv \quad (5.21)$$

Now, the integral-sum on the RHS is essentially the net recombination rate, (*cf.*equation 5.5), times the mean energy released per recombination:

$$\Lambda_{rec} \simeq n^2 x^2 \alpha(T) k_B T \quad . \quad (5.22)$$

The integral is over the electron distribution function, and the sum is over all relevant energy levels.

Can hydrogen cooling alone account for the temperature of an HII region? We observe temperatures $\lesssim 10^4$ K. Now, if the heating is by hydrogen ionization, and the cooling is by hydrogen recombination, our thermal balance would be

$$\Gamma_{uv}(T) = \Lambda_{H,rec}(T) \quad (5.23)$$

Comparing (5.20) and (5.22), we see that this solves to

$$k_B T \simeq \langle h(\nu - \nu_1) \rangle \quad (5.24)$$

(everything about the density and ionization state drops out, note – due to the ionization rate balance). But this is too hot; recall $h\nu_1 = 13.6$ eV. Thus, simple estimates don't work here – we need to include heavy element cooling, which turns out to be stronger, and results in the lower temperatures we see.

For the heavy element cooling rates, we can write schematically (for element X)

$$\Lambda_X = \sum_{j,k} n_{X,j} A_{X,jk} h\nu_{jk} \quad (5.25)$$

where the sum is over all "upper" and "lower" states, and the level populations $n_{X,j}$ must be determined by the methods discussed previously. An important fact to know is that the levels are collisionally excited (but radiatively de-excited, of course). Thus, the net cooling rate $\Lambda_x \propto n_e n_X \propto n^2$, since it is a collisional process.

Thermal balance in the nebula can then be written as

$$\Gamma_{uv}(T) = \Lambda_{rec}(T) + \sum_X \Lambda_X(T) \quad . \quad (5.26)$$

Solutions of this equation determine the temperature of the nebula. From (5.12), and the discussion just above, we see that both sides $\propto n^2$, so that the equilibrium temperature does not depend on the density. The temperature can be found numerically.

This is the big result: the temperature does not depend on the density; only on the details of the ionizing spectrum and coolants. Thus, the temperature of any (photoionized) HII region is $\sim 8000-10,000$ K. This result is independent of the density (nearly) and of the number of stars ionizing the region; it has some dependence on the ionizing spectrum and the abundance of heavy elements, primarily oxygen. Through the level populations it does have a weak dependence on the density.

5.3 The diffuse ISM: multiphase equilibrium

For the general diffuse ISM, things aren't quite so simple. However, Field etal (1969) developed a very nice semi-analytic formulation; here I simplify the algebra[2] by extracting only the dominant terms. We begin by determining what the most important cooling and heating mechanisms are. We start with cooling, which is the simpler of the two, as it only depends on microphyics.

> *LOOKING AHEAD*: The big results here are (i) the general idea of setting up a "heating = cooling" balance for the ISM; and (ii) the result, that the cold and warm phases which coexist in the ISM are thought to be two stable solutions of a "heating = cooling" balance.

5.3.1 Cooling: what dominates here?

The ISM cools by radiation (it's optically thin to many/most photons). The radiation is generated, typically, by collisional excitation of some atom or other, by a free electron. The atom de-excites by radiation, leading to a net energy loss from the system. If we want to find which of all possible line transitions are important for cooling, we can think of several criteria.

- The species must be fairly abundant;
- its excitation energy must be comparable to, or less than, the typical electron thermal energy;
- it must have a large cross section for such excitation;

[2] honestly, folks!

• it must have a high probability to de-excite before another collision occurs;

• and it must generate photons to which the ISM is optically thin.

Given this general argument, we find that two of the most important coolants for the temperature range that describes the cold and warm hydrogen (which is a few $\times 10^2$ to a few $\times 10^3$ K) are particular lines from hydrogen and from carbon. We can approximate the cooling rate of these by saying that the line radiatively de-excites as soon as it is excited; thus, the cooling rate is just proportional to the excitation rate. But the collisional excitation rate is, typically,

$$q_{X;ij} \simeq q_{Xo} T^{-1/2} e^{-h\nu_{ij}/k_B T} \tag{5.27}$$

for excitation from level i to level j of species X. The term q_{Xo} is a numerical factor containing atomic constants.[3] For our two main coolants, the cooling rate can be written,

$$\Lambda_H \simeq n_e n_{HI} \frac{q_H}{T^{1/2}} e^{-h\nu_H/k_B T} h\nu_H \tag{5.28}$$

and

$$\Lambda_C \simeq n_e n_{HI} \frac{n_C}{n_H} \frac{q_C}{T^{1/2}} e^{-h\nu_C/k_B T} h\nu_C \tag{5.29}$$

Here, we are assuming that $x \ll 1$, so that $n_{HI} \simeq n$. We note that the line frequencies are given by $h\nu_C/k_B \simeq 92$ K, and $h\nu_H/k_B \simeq 10^5$ K. Thus, at low temperatures C cooling dominates, and at higher temperatures H cooling dominates.

5.3.2 Heating: by cosmic rays?

As an example of how this can be quantified, consider cosmic ray heating. This may not be the dominant heating; however it is easy to write down analytically (as Field etal did), and therefore provides a useful example of the analysis. In addition, cosmic ray heating is proportional to the local density, just as X-ray heating and photoelectric heating are (and probably as magnetic heating would be, too). Thus, the analysis I present here could be extended to PAH heating by changing the constants; the results for the multiphase ISM should be fairly robust.

In cosmic ray heating, the neutral fraction will be heated by ionization. If $\sigma_{ion}(E)$ is the cross section for ionization by a cosmic ray of energy E, $f_{cr}(E)$ is the density of cosmic rays at energy E, and ΔE is the excess energy the electron comes away with in this ionization, the ionization rate per volume can be written,

$$n_{HI}\varphi_{cr} = n(1-x) \int f_{cr}(E) v(E) \sigma_{ion}(E) dE \tag{5.30}$$

From this, the heating rate due to cosmic ray ionization can be written,

$$\begin{aligned} \Gamma_{cr,ion} &= n(1-x) \int f_{cr}(E) v(E) \sigma_{ion}(E) \Delta E dE \\ &= n(1-x)\varphi_{cr}\langle \Delta E \rangle \end{aligned} \tag{5.31}$$

in direct analogy to (5.20). Numerically, this turns out to be

$$\Gamma_{cr,ion} \simeq 5 \times 10^{-12} \varphi_{cr} n(1-x) \ \text{erg cm}^{-3}\ \text{s}^{-1}$$

Current estimates for φ_{cr}, averaged over the disk, suggest $\varphi_{cr} \simeq 10^{-17}$ s^{-1}.

The ionized fraction of the gas will be heated by Coulomb collisions with the cosmic rays. A single cosmic ray, once it is relativistic, loses energy as a rate $P_C(E) \simeq 6 \times 10^{-19} nx$ erg/s to the ionized component (this number comes from the Coulomb collision rate for relativistic particles). Thus,

$$\Gamma_{cr,C} = \int f_{cr}(E) P_C(E) dE \tag{5.32}$$

To simplify the algebra below, we note that $\Gamma_{cr,C}$ can be related to φ_{cr}, since both involve an integral over $f_{cr}(E)$, the cosmic ray distribution function. We will write this relation as $\Gamma_{cr,C} = \mathcal{A}\varphi_{cr} nx$, where $\mathcal{A}$ contains a mean of $P_C(E)/\sigma_{ion}(E)$, and other (numerical) factors. Finally, we can also write down the ionization state of the gas, due to the cosmic rays:

$$\varphi_{cr} n(1-x) = n^2 x^2 \alpha(T) \tag{5.33}$$

which has limiting solutions for x, just as in (5.7).

5.3.3 Thermal balance and multiphase equilibrium

We will follow the method first presented by Field et al (1969), and will approximate the behavior of the heating and cooling functions to allow us to find an analytic solution for the equilibrium temperatures. This analysis addresses the two cooler phases of the ISM, the

[3] The exponential, and the inverse $T^{1/2}$, arise from an integral of the collision rate, $1/n\sigma(v)v$, integrated over the electron velocity distribution; see if you can justify this for yourself.

cold and warm HI distributions. (The hot, coronal gas is probably heated dynamically, by supernova shocks and/or stellar winds, and also cooled dynamically by its expansion out of the galactic plane; it will not be included in this analysis).

Now, the general thermal balance equation, for heating by cosmic rays and cooling by these two spectral lines, from C and H, is

$$\varphi_{cr} n(1-x)\langle \Delta E\rangle + \varphi_{cr}\mathcal{A} n x = n^2 x(1-x) \times \left[\frac{q_H}{T^{1/2}} e^{-h\nu_H/k_B T} h\nu_H + \frac{n_C}{n_H}\frac{q_C}{T^{1/2}} e^{-h\nu_C/k_B T} h\nu_C\right] \tag{5.34}$$

This equation gives the implicit (n, T) solution, for the density-temperature relation of the equilibrium gas. We show this solution, below; but it seems worth getting some insight into its behavior first. Consider only the low-temperature regime, where we can ignore H cooling, and also ignore $\Gamma_{cr,C}$ (since $x \ll 1$). The balance is then, approximately,

$$n\varphi_{cr}(1-x)\langle \Delta E\rangle \simeq n^2 x(1-x)\frac{n_C}{n_H}\frac{q_C}{T^{1/2}} e^{-h\nu_C/k_B T} h\nu_C \tag{5.35}$$

But from this, using the $x \ll 1$ limit of (5.33), we find that

$$n \propto \alpha(T) e^{2h\nu_C/k_B T} \propto \frac{1}{T^{1/2}} e^{2h\nu_C/k_B T} \tag{5.36}$$

This gives us an approximate analytic version of the (n, T) relationship for $T \sim 10^2 - 10^3$ K. We can find a similar expression for the high-T regions, where H cooling dominates. The net solution looks like the cartoon in Figure 5.2. This solution can also be turned into a (p, T) solution, using $p \propto nT$, also in Figure 5.2.

But this is, then, (almost) the end of the analysis. That is, since we know the pressure of the system – as we do, observationally ($nT \sim$ few $\times 10^3$ cm^{-3} K), we can find the (p, T) solutions which are allowed by this thermal balance. Figure 5.2 illustrates the solutions; in principle anywhere on the (n, T) or (p, T) loci is a solution. We restrict the possibilities by specifying the pressure – equating it to the ISM pressure (which is known). This gives us three possible solutions (as in Figure 5.2). But will all three exist in the ISM?

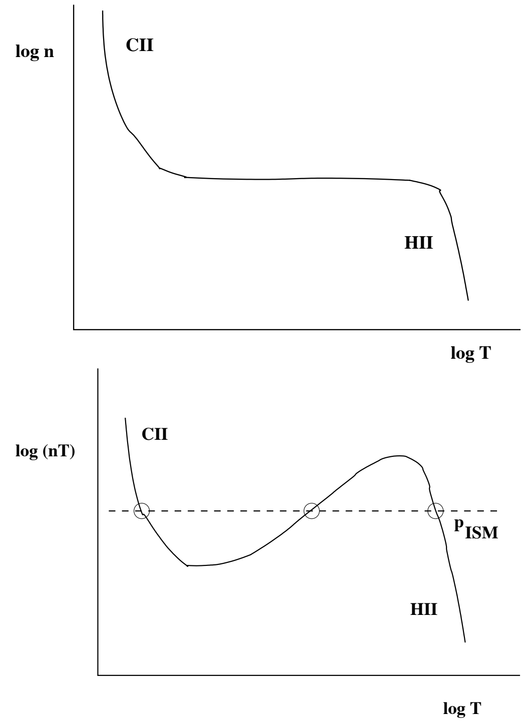

Figure 5.2 Top; sketch of the (n, T) solution from (5.34). The lower temperatures are controlled by CII cooling; the higher temperatures by HII cooling. Any (n, T) pair on this curve is a possible solution; but not all will exist in the ISM. Bottom: the same solution, but now plotting "pressure" nT against T. The dotted line indicates the ISM pressure, which picks out (n, T) solutions that might exist in the ISM. The small circles show three possible solutions; however only the two outer ones are thermally stable. When we put in real numbers, these stable solutions correspond to the *cold* and *warm* phases of the ISM.

5.3.4 Is the thermal balance solution stable?

The final step of the argument is to argue, as Field etal did, that the outer two solutions – $T \sim 10^2$ K and $T \sim$ few $\times 10^3$ K – are the two stable HI phases; and that the middle T solution is unstable. The argument goes as follows. Consider one particular equilibrium state, with density and temperature specified, as well as with external pressure fixed. For instance, look the middle T solution in Figure 5.2. Consider a slight compression of this state, in which the temperature drops a little bit (this is required from the (n, T) solution in this region). But the pressure must also drop in the perturbation (from the (p, T) curve); so the larger external pressure will compress the perturbation still further – and the density will drop still more, and it will cool faster . . . and so on. That is, this system is *thermally unstable*; a slight compression will collapse and cool, and a slight rarefaction will expand and reach higher temperatures. The timescale for this runaway $\sim t_{cool} \sim k_B T/\Lambda_{net}(T)$, as always; for the ISM, this is a short time, so the instability will develop rapidly.

Conversely, for the outer two solutions, the slope of the (p, T) curve is reversed, so that – say, a small compression will reach higher pressures, and expand back to its original state (or vice-versa for a small rarefaction). Thus, the two outer states should be stable, and are believed to represent the two cool phases of the ISM.

More formally, Field etal (1969) worked with the net cooling function, $\mathcal{C} = \Lambda - \Gamma = n^2\mathcal{L} - n\mathcal{H}$. Again, the cooling term is $\Lambda = n^2\mathcal{L}$ as in Figure 5.1. The heating term is written as $\Gamma = n\mathcal{H}$ to highlight the linear dependence on density n. They showed that the system is unstable if

$$\left.\frac{\partial \mathcal{C}}{\partial T}\right|_p = \left.\frac{\partial \mathcal{C}}{\partial T}\right|_n - \frac{n}{T}\left.\frac{\partial \mathcal{C}}{\partial T}\right|_T < 0 \qquad (5.37)$$

Now, using $\mathcal{C} = 0$ (which means we start in thermal balance, before we perturb the system) in (5.37) turns this condition into

$$\left.\frac{\partial \mathcal{C}}{\partial T}\right|_p = n^2\frac{d\mathcal{L}}{dT} - n^2\frac{\mathcal{L}}{T} < 0 \qquad (5.38)$$

as the condition for instability. This can be rewritten as

$$\frac{d\ln\mathcal{L}}{d\ln T} < 1 \qquad (5.39)$$

for the instability condition; the system is stable otherwise. This makes it clear that the *slope* of the cooling curve in Figure 5.1 controls the thermal stability of the system. The middle phase of Figure 5.2 lies at a T where $\mathcal{L}$ is a very slow function of T; so this phase is unstable. The outer two phases of Figures 5.2 lie at T where $\mathcal{L}$ is a steep function of T; thus these phases are thermally stable.

References

Most of the formal material here can be found in

• Spitzer (*Physics of the Interstellar Medium*;

but the thermal stability details come from the original Field, Goldsmith & Habing paper (1969 ApJ), and I've also pulled more recent arguments (on heating, cooling mechcanisms) from the current literature.

Key points

• The ISM cooling curve (and what's inside it);

• Photonionization heating "always" gives $\sim 10^4$ K;

• Stromgren spheres – what they are, what size they are;

• General ISM thermal balance and why we have the two cooler phases;

• Why does ISM thermal balance *not* explain the hot (coronal) phase?