6 Dynamics of the ISM: energetics & shocks

Last term we worked with the mass and momentum conservation laws of fluid dynamics, and applied them to various problems. We now return to fluid dynamics, and consider the physics of shocks in fluid flow. But we can't do that until we look at the third important conservation law, energy conservation in fluids.

6.1 Fluids: energetics

We must consider two forms of energy: the kinetic energy density of bulk flows, $\rho v^2/2$, and internal energy density. The latter is the energy contained in random (thermal) motions of the particles. We will work with the internal energy per unit mass, $e = \frac{1}{\gamma-1}\frac{p}{\rho}$. For a sub-relativistic, monatomic gas, for instance, $e = \frac{3}{2}\frac{k_BT}{m}$. The net energy in our volume V is $\int_V \rho(e+\frac{1}{2}v^2)dV$. The net rate of change of this energy from intrinsic changes and from flows is

$$\int_V \frac{\partial}{\partial t}\left[\rho\left(e+\frac{1}{2}v^2\right)\right]dV + \int_V \nabla\cdot\left[\rho\mathbf{v}\left(e+\frac{1}{2}v^2\right)\right]dV, \tag{6.1}$$

where we have used the divergence theorem to convert a surface integral to a volume integral as in Chapter 4 of the 425 notes. This net energy change must be accounted for by (a) work done by an external acceleration $\mathbf{f}$, which is often taken to be gravity; (b) work done by the external pressure; (c) direct energy gains or losses, most commonly direct heating (by photons or cosmic rays, say), or radiative losses, as in chapter 5. These three energy-change factors are

$$\int_V \rho\mathbf{f}\cdot\mathbf{v}dV - \int_A p\hat{\mathbf{n}}\cdot\mathbf{v}dA + \int_V(\Gamma-\Lambda)dV \tag{6.2}$$

Again we can use the divergence theorem to convert the pressure work term to a volume integral, and we can derive one version of the differential energy conservation law:

$$\frac{\partial}{\partial t}\left[\rho\left(e+\frac{1}{2}v^2\right)\right] + \nabla\cdot\left[\rho\mathbf{v}\left(e+\frac{1}{2}v^2\right)\right] = \rho\mathbf{f}\cdot\mathbf{v} - \nabla\cdot(p\mathbf{v}) + \Gamma - \Lambda \tag{6.3}$$

The forms we derived for the mass and momentum conservation equations are pretty standard. However, there does not seem to be one standard form for the energy conservation equation; rather, one uses the form that works best in a given application. Therefore, at the expense of a little algebra, we will look at several alternate forms of (6.3).

First, with the help of the continuity equation[1] we can separate out the $\partial\rho/\partial t$ and $\nabla\cdot(\rho\mathbf{v})$ terms in (6.3), we find

$$\rho\frac{\partial}{\partial t}\left(e+\frac{1}{2}v^2\right) + \rho\mathbf{v}\cdot\nabla\left(e+\frac{1}{2}v^2\right) = \rho\mathbf{f}\cdot\mathbf{v} - \nabla\cdot(p\mathbf{v}) + \Gamma - \Lambda \tag{6.4}$$

which is one alternate form that we will use again. We can isolate the rate of change of e, from (6.4), by subtracting $\mathbf{v}\cdot$(the momentum conservation equation), giving

$$\rho\frac{\partial e}{\partial t} + \rho\mathbf{v}\cdot\nabla e = -p\nabla\cdot v + \Gamma - \Lambda \tag{6.5}$$

In this expression, we can see that the rate of change of the internal energy depends explicitly on compression work ("pdV" work), and on the net heating and cooling rates.

Yet another common form of the energy equation is found by defining the *convective*, *total* or *Lagrangian* derivative,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla \tag{6.6}$$

With this, we can use the continuity equation to write $\nabla\cdot\mathbf{v}$ in terms of the density derivatives, and use $e = \frac{1}{\gamma-1}\frac{p}{\rho}$ to write

$$\frac{\rho}{\gamma-1}\frac{D}{Dt}\left(\frac{p}{\rho}\right) - \frac{p}{\rho}\frac{D\rho}{Dt} = \Gamma - \Lambda \tag{6.7}$$

or, if we collect the p and ρ derivatives separately, we get

$$\frac{D}{Dt}\left(\frac{p}{\rho^\gamma}\right) = (\gamma-1)(\Gamma-\Lambda) \tag{6.8}$$

which is the last of our alternate forms of the energy equation.

This last form allows us to consider a couple of important limits. The first is the *adiabatic limit*. If $\Gamma-\Lambda=0$, so that there is no net gain or loss of energy to the system, (6.8) shows that

$$\frac{p}{\rho^\gamma} = \text{constant} \tag{6.9}$$

[1] You saw this last term, in chapter 4 of the P425 notes. One form is

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0$$

which is the usual adiabatic law (the consequence of there being no gain or loss of heat from a system). The second limit is the *isothermal limit*. A good many calculations assume T = constant, which simplifies things enormously. From (6.5), we see that

$$p\nabla \cdot \mathbf{v} = \Gamma - \Lambda \tag{6.10}$$

is the condition that must be satisfied if T (or e) is constant.

6.2 Supersonic flow and shock fronts

In P425, we found a characteristic signal speed in a gas, namely the sound speed, c_s. This is a critical finding: because this is the speed at which a perturbation propagates, "information" about changes in the flow can only propagate at c_s. Figure 6.1 illustrates this, in a moving flow.

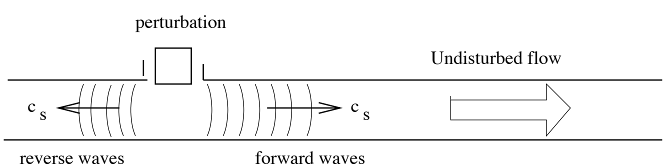

Figure 6.1 Illustrating signal propagation in a flow. A perturbation "whacks" the pipe at one spot; the information that this has happened travels upstream and downstream at c_s, *relative to the flow*. Following Thompson figure 8.6.

Consider, then, gas moving at a speed greater than the sound speed; if follows that information cannot propagate upstream. This means the gas generally cannot adjust smoothly to changes in the ambient or boundary conditions, but rather must adjust instantaneously - creating a discontinuity in the flow. Such a discontinuity is a *shock*. Examples are bow shocks around supersonic aircraft, or around the planets (since the solar wind is supersonic); or standing shocks, such as where supersonic flow runs into a zero-velocity surface (for instance, at the end of a radio jet, where it runs into the ambient plasma).

We treat a shock as an infinitely thin discontinuity in a flow. The true width of the shock is determined by collision processes within the fluid, and by assumption these operate on scales much smaller than those described by the fluid equations. The intent is to derive "jump conditions" – to use the basic conservation laws to derive relations between the fundamental variables (ρ, p, T, v) upstream and downstream of the shock. Let "1" describe upstream, and "2" describe

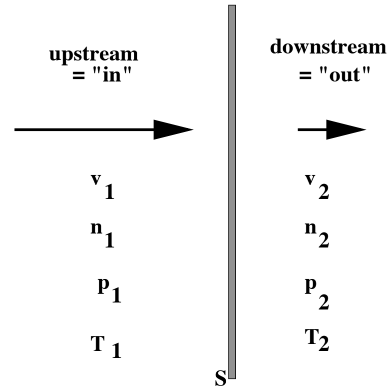

Figure 6.2 A close-up look at a shock, S, in a frame in which the shock is at rest. The incoming fluid is labelled with subscript "1"; outgoing with subscript "2". The incoming fluid must be supersonic, $v_1 > c_{s1}$. The outgoing fluid is slower ($v_2 < v_1$), denser, $n_2 > n_1$), and probably hotter ($T_2 > T_1$, for an adiabatic shock); the incoming kinetic energy is converted to heat when the shock decelerates the flow.

the downsteam flow, as seen in a frame moving with the shock (as in Figure 6.2). Let the *Mach number* be $\mathcal{M} = v/c_s$ (generally defined for upstream flow). Consider steady, one-dimensional flow, with no external forces, and with no net external heating or cooling. Referring back to the footnote on the prevous page, the continuity equation for the fluid in steady state becomes $\nabla \cdot (\rho \mathbf{v}) = 0$. If we integrate this over a small, Gaussian surface enclosing some part of the shock plane, we get

$$\rho_1 v_1 = \rho_2 v_2 \tag{6.11}$$

This is, of course, simply mass conservation: the flux in (per area) must equal the flux out. The force equation for the fluid is

$$\nabla \cdot (\rho \mathbf{v}\mathbf{v}) + \nabla p = 0 \tag{6.12}$$

(again, go back to your P425 notes, where this form was presented implicitly, as equation (4.3), but not elaborated on). Integrating this across the shock face, we get

$$\rho_1 v_1^2 + p_1 = \rho_2 v_2^2 + p_2 \tag{6.13}$$

These two equations are general. The energy equation is generally applied in either the adiabatic or isothermal limits.

6.2.1 Adiabatic shocks

The form (6.3) of the energy equation, with some algebra (always!) and applied to our conditions here, gives us

$$\frac{\gamma}{\gamma-1}p_1v_1 + \frac{1}{2}\rho_1v_1^3 = \frac{\gamma}{\gamma-1}p_2v_2 + \frac{1}{2}\rho_2v_2^3$$

Factoring out ρv from each side, and using (6.13), this can be written

$$\frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}v_1^2 = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}v_2^2 \qquad (6.14)$$

Now, this can be combined with (6.11) and (6.13), to express three post-shock quantities, ρ_2, v_2 and p_2, in terms of their pre-shock counterparts. This solution is:

$$\begin{aligned}\frac{\rho_1}{\rho_2} &= \frac{\gamma-1}{\gamma+1} + \frac{1}{\mathcal{M}^2}\frac{2}{\gamma+1}\\ \frac{p_2}{p_1} &= \frac{2\gamma\mathcal{M}^2-(\gamma-1)}{\gamma+1}\\ \frac{v_2}{v_1} &= \frac{\gamma-1}{\gamma+1} + \frac{1}{\mathcal{M}^2}\frac{2}{\gamma+1}\end{aligned} \qquad (6.15)$$

When $\mathcal{M} \gg 1$, these equations simplify, to

$$\begin{aligned}\frac{\rho_1}{\rho_2} &= \frac{\gamma-1}{\gamma+1}\\ \frac{p_2}{p_1} &= \frac{2\gamma\mathcal{M}^2}{\gamma+1}\\ \frac{v_2}{v_1} &= \frac{\gamma-1}{\gamma+1}\end{aligned} \qquad (6.16)$$

In particular, if $\gamma = 5/3$, we find $\rho_2/\rho_1 = v_1/v_2 = 4$ (this is often quoted as the strong shock limit). And, in this limit, the temperature jump is $T_2/T_1 = 5\mathcal{M}^2/16$, giving $k_BT_2 = 3mv_1^2/32$; the upstream kinetic energy is converted to internal energy in an adiabatic shock.

6.2.2 Isothermal shocks

The energy equation is this case is simple: $T_2 = T_1$ by assumption. The possibility of an isothermal shock depends on the cooling times. We would expect a general shock to have a structure as in Figure 6.3. The gas passing through the shock is initially heated, by adiabatic compression, and suffers a moderate density jump. This hotter gas (assuming an optically thin situation), can then cool by radiation, and while cooling the gas travels a distance $\sim v_2t_{cool}$. This "cooling distance" thus measures the effective width of the transition to an isothermal shock – assuming that some heating/cooling balance, as we described for the ISM, maintains the upstream and far-downstream temperature at T_1.

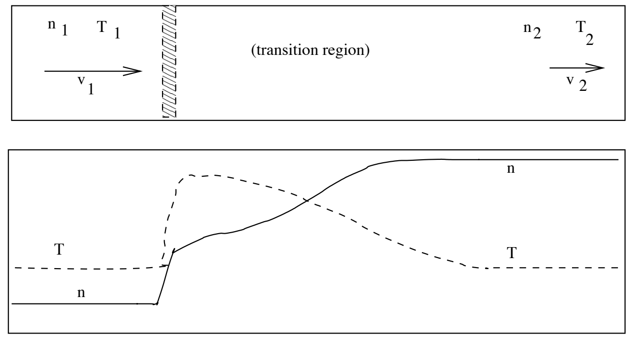

Figure 6.3 Schematic diagram of a radiating shock. In the upper diagram, the fluid is coming in from the left. At x_1 it enters the nonradiative shock. This is followed, to the right, by the transition region, where the temperature drops as the gas cools by radiation. The changes of density and temperature are shown schematically in the lower figure. Following Spitzer figure 10.1.

Combining $T_2 = T_1$ with (6.11) and (6.13), to express isothermal jump conditions,

$$\begin{aligned}\frac{\rho_2}{\rho_1} &= \mathcal{M}^2\\ \frac{v_2}{v_1} &= \frac{1}{\mathcal{M}^2}\\ \frac{p_2}{p_1} &= \mathcal{M}^2\end{aligned} \qquad (6.17)$$

Thus, the compression factor, and deceleration factor, can be much higher for an isothermal shock than for an adiabatic one.

6.2.3 Magnetized shocks

The previous analysis ignored the magnetic field. This is probably too naive, as we know the ISM is magnetized. We can understand the effect of a shock on the field by considering flux freezing. Refer to the left part of Figure 6.4. In this case, the field is parallel to the shock face. In this geometry, the field is tied to the gas by flux freezing; thus the field behind the shock will be increased, in the same amount as the gas is compressed. By comparison, consider the right part of the figure, in which the shock propagates along the field lines. In this case, the density jump will not affect the magnetic field (why? can you see how this is consistent with flux freezing?).

6.2.4 Oblique shocks

Finally, what if the shock face is not perpendicular to the flow? The answer is qualitatively simple, and can be readily understood by thinking about an oblique in-

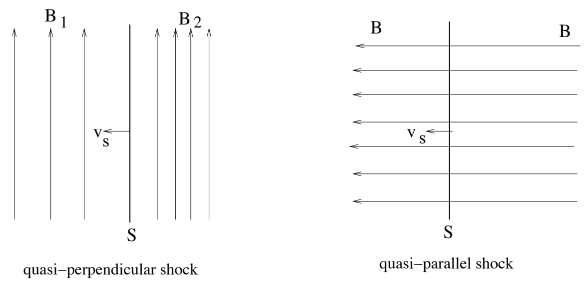

Figure 6.4 Schematic picture of magnetized shock, in two important limits The shock speed relative to the medium is v_s. Left, quasi-perpendicular shock (note $v_s \perp B$): flux freezing increases the post-shock field, by a factor $B_2/B_1 = \rho_2/\rho_1$. Right, quasi-parallel shock; the gas flows along the field lines without perturbing the field.

coming velocity (**w** in the Figure 6.5, which illustrates the geometry) in terms of its components parallel and perpendicular to the shock face. The component perpendicular to the shock face is decelerated, just as in the normal-shock results above. The component parallel to the shock face, however, is not affected (query to the reader: why not?). Thus, the net velocity bends *toward* the shock face.

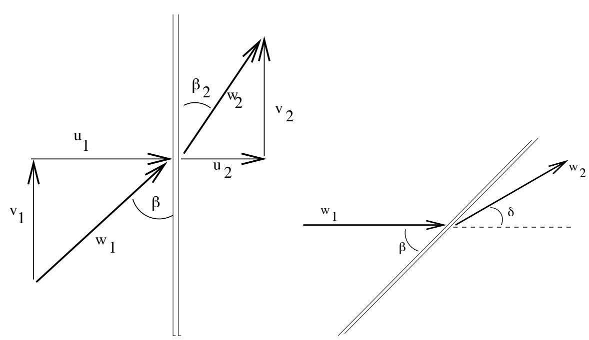

Figure 6.5 Oblique shocks. The velocity bends towards the shock face, as shown in the right figure; this can be understood by considering the effect of the shock on the velocity components, as in the left figure.

Where do we expect to deal with oblique shocks? In just about any 2D situation ... a good example is the terrestrial bow shock, where the supersonic solar wind encounters the earth. The flow coming in bends toward the shock normal, and thus is deflected around the earth. The jump conditions (extensions of 6.15 or 6.17) can be quite complicated algebraically; we won't deal with them in this course.

Key points

- Energy conservation: adiabatic and isothermal limits
- Shock fronts: jump conditions across the shock
- Shock fronts: adiabatic and isothermal limits
- Effects of B field or oblique angle: qualitative