7 Stellar Winds & Supernovae Remnants

Now let's look at some supersonic flow situations which involve shocks. You remember that we worked with smooth transonic flows last term – for instance the solar wind. These are rare; it's very easy to form shocks when supersonic flows are decelerated or bent by their surroundings. In this chapter we'll work with 2-1/2 examples: stellar wind shocks, and two types of supernova remnants.

7.1 Stellar winds and the surrounding ISM

We worked with smooth, transonic stellar wind flow last term ... and found a solution in which the outer regions of the wind flow are supersonic. But this can't carry on forever. At some point the wind flow must run into the ambient ISM. We expect the deceration to lead to an outer shock in the wind; what are the details? By way of review, I'll repeat the basics of the inner-wind solution here.

7.1.1 The basic solution

• Mass conservation in a steady, spherical flow is $\rho v r^2 = \text{constant}$; or,

$$\frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{v}\frac{dv}{dr} + \frac{2}{r} = 0 \tag{7.1}$$

while the momentum equation becomes in this case (noting that gravity from the central star is important),

$$\rho v \frac{dv}{dr} + \frac{dp}{dr} = -\rho \frac{GM}{r^2} \tag{7.2}$$

Writing $dp/dr = c_s^2 d\rho/dr$, these two equations combine to give the basic wind equation,

$$\left(v - \frac{c_s^2}{v}\right)\frac{dv}{dr} = \frac{2c_s^2}{r} - \frac{GM}{r^2} \tag{7.3}$$

This does not have analytic solutions over the whole range of r. However, we can learn quite a bit about the nature of the solutions simply by inspection of (7.3), as follows.

• First, the left hand side contains a zero, at $v^2 = c_s^2$. If we want to consider well-behaved flows, that is to say those in which the derivative dv/dr does not blow up, then the right hand side of (7.3) must go to zero at the same point. This defines the condition that must be met at the sonic point:

$$v^2 = c_s^2 \qquad \text{at} \qquad r = r_s = \frac{GM}{2c_s^2} \tag{7.4}$$

Whether or not a particular flow satisfies this condition depends on the starting conditions, such as with what velocity and temperature it left the stellar surface, and also what the boundary conditions at large distances are. If it does not start in such a way to satisfy this condition, it either stays subsonic (corresponding to finite pressure at infinity), or cannot establish a steady flow.

• Further, the solution beyond the sonic point depends on the temperature structure of the wind. The only solutions with $dv/dr > 0$ for $r > r_s$ are those for which $c_s^2(r)$ drops off more slowly than $1/r$; it is only these for which the right-hand side stays positive. In the case of an isothermal wind, with $c_s^2 = \text{constant}$, (7.3) can be solved in the limit $r \gg r_s$:

$$v^2(r) \simeq 4c_s^2 \ln r + \text{constant} \tag{7.5}$$

Thus, the wind will be supersonic, by a factor of a few, as $r \to \infty$. The question of how the solar wind manages to stay nearly isothermal is not solved; it is probably due to energy transport by some sort of waves (MHD or plasma waves, for instance) which are generated in the photosphere and damped somewhere far out in the wind.

7.1.2 The outer shock

The pressure in this supersonic wind is dropping with radius (since $\rho \propto 1/vr^2$, with v being only slowly varying; thus $p \propto \rho T \propto 1/r^2$, approximately, in an isothermal wind). Therefore, when the wind pressure is close to the ISM pressure, the wind must slow down. At this outer boundary, we expect some sort of shock transition, since the wind is supersonic. Past this shock, the hot, shocked wind-gas will expand into the ISM (at about its own sound speed, to start); as long as this expansion is supersonic relative to the ISM, the expanding hot gas will drive a "snowplowed" shell of ISM, and a second shock, out into the ISM.

A cartoon of this region, at some point in time, would be that in Figure 7.1. Let region "a" be the wind; S_1 be the inner shock; region "b" be the wind-gas which has been through the shock; C be the contact surface between the wind and the ISM; region "c" be the shocked ISM; and S_2 be the outer shock (moving into the ISM). We expect S_1 to be an adiabatic shock (since the wind is probably hot and low density, and thus will have a long cooling time); region "b" will contain hot, shocked wind, with $T_b \sim \frac{3}{16}\frac{mv_{wind}^2}{k_B} \sim \text{several} \times 10^7$ K (noting that $m = \frac{1}{2}m_p$ is the mean mass per particle if region "b" is fully ionized). The outer shock will probably be isothermal, since the ISM is denser and cooler than the wind. Thus, the shocked ISM will be in a thin shell, containing all of the original ISM that lay between S_2 and the star.

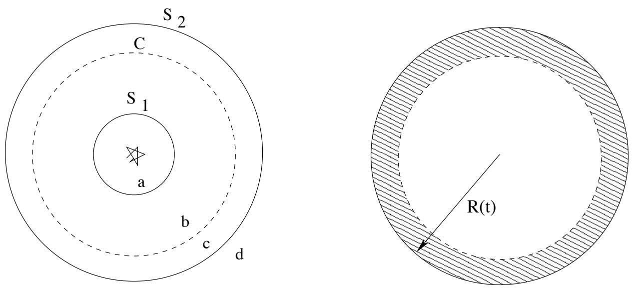

Figure 7.1 Cartoon of the structure of a stellar wind, and its interaction with the ISM. Left: the shock structure within the wind. Right: The outer shell of dense, snowplowed ISM. From Dyson & Williams figures 7.3 and 7.4.

To start, consider the position of this shell, $R(t)$, as a function of time. We start with a force equation. The mass is the ISM shell is the mass that was originally within $R(t)$: $\frac{4\pi}{3}\rho_o R^3(t)$. The force acting on the shell is just the pressure behind it, which drives it outward:

$$\frac{d}{dt}\left(\frac{4\pi}{3}R^3\rho_o\frac{dR}{dt}\right) = 4\pi R^2 p_b \qquad (7.6)$$

if p_b is the pressure, in the outer part of region "b", which acts on the shell. Next, we need an equation for $p_b(t)$. The energy within region "b" – which we will take to be nearly all of the volume within $R(t)$ – is $\simeq 2\pi p_b R^3(t)$; the rate of change of this energy is given by the difference between the input (from the wind) and the "pdV" work done by the expansion:

$$\frac{d}{dt}\left(2\pi R^3 p_b\right) = \dot{E}_{wind} - p_b\frac{d}{dt}\left(\frac{4\pi}{3}R^3\right) \qquad (7.7)$$

if $\dot{E}_{wind}$ is the energy input rate from the wind, assumed constant. These two equations can be combined, for instance by eliminating p_b, to get

$$aR^4\frac{d^3R}{dt^3} + bR^3\frac{dR}{dt}\frac{d^2R}{dt^2} + cR^2\left(\frac{dR}{dt}\right)^3 = \frac{3}{2\pi}\frac{\dot{E}_{wind}}{\rho_o} \qquad (7.8)$$

where a, b, c are numerical constants (that you'll find in the homework). Noting that each term on the left hand side has the same dependence on R and t ($\sim R^5 t^{-3}$), we can try a power law solution, $R(t) = At^{\alpha}$. A small bit of algebra tells us that $\alpha = 3/5$ is the allowed solution, and we can also find an expression for A. Thus, the solution is

$$R(t) = 0.76\left(\frac{\dot{E}_{wind}}{\rho_o}\right)^{1/5} t^{3/5} \qquad (7.9)$$

and

$$v(t) = \frac{dR}{dt} = 0.46\left(\frac{\dot{E}_{wind}}{\rho_o}\right)^{1/5} t^{-2/5} \qquad (7.10)$$

Thus, the shell decelerates with time, as it ought to if it is picking up more and more ISM. One can then use these, and (7.6), to work out the pressure acting on the outer shell:

$$p_b(t) = \frac{7}{25}A^2\rho_o t^{-4/5} \qquad (7.11)$$

so that the outer pressure drops with time. Finally, one can also work out the kinetic energy of the shell, which must give the energy input to the general ISM from the wind. This turns out to be

$$\frac{2\pi}{3}R^3\rho_o\left(\frac{dR}{dt}\right)^2 \simeq 0.2\dot{E}_{wind}t \qquad (7.12)$$

so that about 20% of the wind energy goes to the ISM. (The rest goes to heating the bubble, and to "pdV" work).

7.1.3 What about the inner shock?

The location of the inner shock, S_1, is determined by a combination of the jump conditions, applied at S_1, and the pressure at the outside of the region, p_b, as follows.

• The jump conditions, at S_1, are $\rho_{sb} = 4\rho_{sa}$, if "sb" and "sa" subscripts refer to postshock (region "b") and preshock (region "a"), respectively. Also, $v_{sb} = v_{sa}/4$ – here we assume $\mathcal{M} \gg 1$, the strong shock limit, and also an adiabatic shock. At the shock, momentum conservation[1] tells us that $\rho v^2 + p$ is conserved; thus,

$$p_{sb} = \frac{3}{4}\rho_{sa}v_{sa}^2 + p_{sa} \simeq \frac{3}{4}\rho_{sa}v_{sa}^2$$

where we have used the fact that $v_{sa} \gg c_s$ in the wind region.

• In region "b", where we can ignore gravity (compared to the internal energy, $\frac{3}{2}p$), the momentum equation is

$$\rho v\frac{dv}{dr} + \frac{dp}{dr} \simeq 0$$

[1] Check back to chapter 4 from last term, P425; equation 4.4.

and, since $v \ll c_s$ in most of region "b", $\rho \simeq$ constant, and we have $\frac{1}{2}\rho v^2 + p \simeq$ constant. Now – we know that v drops from $v_{sb} = v_{sa}/4$, at S_1, to $v_{s2} \propto t^{-2/5} \ll c_{s,sb}$ at S_2 (from the shock conditions). Thus, we connect the conditions just past S_1 to the conditions at S_2 (remembering that we called the pressure in region "b" at S_2, p_b; ugly notation, I agree!),

$$p_b \simeq p_{sb} + \frac{1}{2}\rho_{sb} v_{sb}^2 = \frac{7}{8}\rho_{sa} v_{sa}^2$$

Thus, p_b, at S_2, is set by the dynamic pressure at S_1. If p_b is also set by external conditions – say the ISM pressure – then the location of S_1 must be where $\rho_{sa} v_{sa}^2$ satisfies the above condition.

• But we can find this location. We note, again, that the dynamic pressure in the wind region, $\rho v^2 \propto \dot{M} v / r^2$ drops approximately as $1/r^2$ (since v is slowly varying). Thus, the behavior of the dynamic pressure within the wind region is fixed by the basic wind solution. Thus, the shock S_1 must form where ρv^2, as determined by the wind solution, matches $\sim \frac{8}{7} p_b$.

7.2 Supernova remnants

First, set the stage: a star explodes. You probably recall the basic picture: stars meet a violent death about 4-5 times per century in a galaxy the size of ours. There are two possible types of supernovae. **Type I supernova** (more correctly Type Ia) arise from the explosion of a white dwarf in a close binary system, presumably initiated by sudden mass transfer from the companion which leads to a thermonuclear reaction in the dwarf star.[2] **Type II supernovae** come from evolved, massive stars in which nuclear burning has run out of fuel. The stellar core collapses inwards and bounces, producing the explosion.

For the purposes of these notes, both types of SNe result in very similar remnants. The dynamics of the ejected material will be sightly different at very early times, due to the differing local environments; but after a short time, both can be treated similarly. That is the approach we will take here. Think of an instantaneous release of energy ($E \sim 10^{51}$ ergs) from a point source, in ambient gas of density ρ_o. The energy released will heat the gas near the explosion to very high temperature and pressure, driving an expansion. This expansion will be very supersonic, setting up a spherical shock wave moving into the surroundings and sweeping up gas as it goes.

This picture is similar to our previous model of an expanding stellar wind bubble; but different in some respects. First, obviously, is the δ-function nature of the explosion. We expect that to change details, for instance the power-law evolution of the radius of the shock. In addition, we have to consider the radiative cooling rate at the outer shell. With stellar winds, we argued that the outer shock is radiative (isothermal); this is justified (after the fact) by the high densities and slow expansion speeds of the wind system. For SNR, however, the situation is different. They can occur in lower density regions (for instance an HII region?), and have much higher explosion speeds (thus much hotter post-shock temperatures). We must consider two phases, then: (a) an early **energy-conserving phase**, during which radiative losses are unimportant, and (b) a later **snowplow phase**, in which the shell becomes dense and cool, and the remnant evolves by momentum conservation.

7.2.1 Early: energy conserving (Sedov) phase.

The remnant in this stage can be thought of as a large hot bubble, filled with ambient gas that has been through the outer, strong shock. Again, let the radius of the outer shock be $R(t)$. In this case, both the internal and kinetic energies per mass are given by

$$e = \frac{1}{2}v^2 = \frac{9}{32}\dot{R}^2 \qquad (7.13)$$

This is a direct post-shock result, and holds in a fixed reference frame – one in which the shock is advancing into a medium at rest. We will also assume that gradients within the bubble are small so that (7.13) holds throughout. (This latter is an OK, but not wonderful, assumption – *cf.* Figure 7.2.) The total energy is then

$$E_{tot} = \frac{4\pi}{3} R^3 \rho_o \left(e + \frac{1}{2}v^2 \right) = \frac{3\pi}{4}\rho_o R^3 \dot{R}^2 \qquad (7.14)$$

But now, we require $E_{tot} = E_{SN}$, the input energy from the SN. We thus have an equation of motion for the shock:

$$R^3 \dot{R}^2 = \frac{4}{3\pi}\frac{E_{SN}}{\rho_o} \qquad (7.15)$$

This solves to

$$R(t) = \left(\frac{25}{3\pi}\right)^{1/5} \left(\frac{E_{SN}}{\rho_o}\right)^{1/5} t^{2/5} \qquad (7.16)$$

[2]Current work splits this hair. The distinction between Type I and Type II SNe was originally based on the presence, or absence, of strong hydrogen lines in the explosion spectra. Originally, all stars without H lines were thought to be explosions of white dwarfs; now I gather Type Ib and Ic are identified with core collapse.

and

$$V(t) = \dot{R}(t) = \frac{2}{5}\left(\frac{25}{3\pi}\right)^{1/5}\left(\frac{E_{SN}}{\rho_o}\right)^{1/5} t^{-3/5} \tag{7.17}$$

Compare these to (7.9, 7.10) for a stellar wind: note the different functional dependence on time.

To go further, the basic fluid equations can be used to determine the structure of the interior of the hot bubble. It is the well-known Sedov-Taylor solution. I do not reproduce it here, but show the results (which must be done numerically) in Figure 7.2.

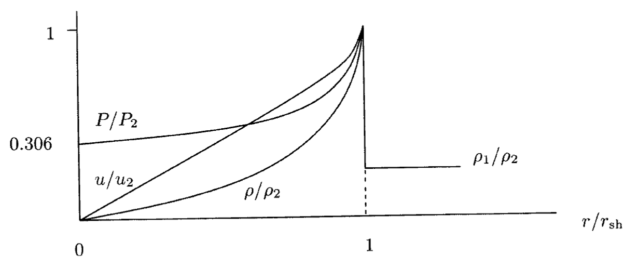

Figure 7.2 Solution for the interior structure of the hot bubble in the energy-conserving phase of the SNR. The radius is scaled to $r_{sh} = R(t)$ in our notation; this solution preserves its functional shape as the remnant expands. From Shu Fig. 17.3.

7.2.2 Late: momentum conserving (snowplow) phase.

At some point the outer shell will cool; as its pressure drops it will be compressed by the hot, expanding inner gas. This is reminiscent of the outer shell in the wind case, but with differences. Here, the dense shell contains only a small fraction of the ISM which was initially inside $R(t)$. In addition, the interior hot bubble is not receiving ongoing energy input, so the pressure on the outwards shell is dropping quickly with time.

The basic analysis of this late-time remnant, then, is based on the simple assumption of momentum conservation. Say the remnant enters this phase at some time t_o, when it has radius R_o and velocity $\dot{R}_o$. Conservation of momentum has,

$$\frac{4\pi}{3}R^3\rho_o\dot{R} = \text{constant} = \frac{4\pi}{3}R_o^3\rho_o\dot{R}_o \tag{7.18}$$

and this integrates to

$$R(t) = R_o\left[1 + 4\frac{\dot{R}_o}{R_o}(t - t_o)\right]^{1/4} \propto t^{1/4} \tag{7.19}$$

and

$$V(t) = \dot{R}(t) = R_o\left[1 + 4\frac{\dot{R}_o}{R_o}(t - t_o)\right]^{-3/4} \propto t^{-3/4} \tag{7.20}$$

where the last proportionality statements are valid for $t \gg R_o/\dot{R}_o$. Thus, comparing this to (7.16,7.17) shows that in the momentum-conserving phase, the remnant expands more slowly . . . as one would expect, right?

Dyson & Williams present some numbers. The Sedov phase has $R(t) \simeq 3.6 \times 10^{-4} t^{2/5}$ pc, and $V(t) \simeq 4.4 \times 10^9 t^{-3/5}$ km/s (if t is in seconds). Strong cooling typically takes over for $\dot{R}_o \sim 250$ km/s, giving $R_o \sim 24$ pc and an age $\sim 30 \times 10^4$ years. At that point, about $1400 M_\odot$ of ISM has been swept up – much larger than the mass initially ejected (something like $4M_\odot$). Thus, we are looking almost entirely at the dynamics of the ISM which was dramatically heated in the star's explosion.

7.3 Plerions, a.k.a. pulsar wind nebulae

The preceding section discussed the "classical" picture of supernova remnants, in which energy is injected *at one instant* into the ambient ISM. Many galactic SNR are well described by this model. But not all: in some cases the exploding star leaves behind an active pulsar as its remnant. We now know that many (most? all?) pulsars drive a relativistic wind out from the star. The wind acts as a source of mass and energy, filling the interior of the SNR with hot (or relativistic) plasma. These "filled" SNR used to be called *plerions* (mostly in the SNR community); these days they are being called *pulsar wind nebulae* (in the ISM/X-ray community). As far as I know the two terms refer to the same type of object.

We have direct evidence of pulsar winds. For older pulsars (those not currently within SNR), we see structures in the nearby ISM which are clearly *bow shocks* associated with the star's high-speed motion through the ISM. From the standoff distance of the bow shock we can estimate the wind energy, and compare it to standard models of the pulsar. For young pulsars (those still within their SNR), recent CHANDRA images directly reveal the outflow from the pulsars (at this point there is a good handful of such images; the Crab and Vela pulsars are the most famous). These outflow are complex: they show *jets*, which presumably come out along the star's rotation axis (this is the only symmetry axis in the system), and *equatorial winds*, which probably arise from the combined effects of the star's strong magnetic field and its rapid rotation.

When a relativistic wind hits the local ISM (which may be the ejecta from the SN), we expect strong shocks to form. Such shocks may be effective at accelerating particles in the local plasma to relativistic energies (we will discuss this in detail later in the course). Thus, the material which has been through the shock may contain a large fraction of relativistic particles – which could, for instance, maintain the nonthermal emission from the surrounding nebula (Crab or Vela). The dyanmics of such a system – assuming it's close to spherically symmetric – are of course very similar to the dynamics of a stellar wind hitting the ISM.

Key points

• Solar-wind solutions (from last term);

• The outer wind shock and getting its location from dimensional analysis;

• Supernova remnants, Sedov and snowplow phases;

• Plerions, what they are, how they work qualitatively.