8 Relativistic particles in astrophysics

Based on our knowledge of cosmic rays and, less directly, of the relativistic electrons which radiate in synchrotron sources, our general picture is of particles which are highly relativistic – that is, with $\gamma = E/mc^2 \gg 1$ – but which are also tied to the background, thermal plasma in which they find themselves. The distribution function of these particles is found to be a power law, rather than a Maxwellian; thus, these particles cannot have had time to thermalize (in the two-body sense). In this section, we will consider the dynamics of these particles and how they are tied to the background plasma). Later on we'll address how they may be accelerated.

8.1 Recap: basics for relativistic particles

This section is just to remind you of some tools that you saw last term.

First, basic special relativity. The total total energy of a relativistic particle is given by $E^2 = p^2c^2 + m^2c^4$; we also have the definition $E = \gamma mc^2$, where $\gamma^2 = 1/(1-\beta^2)$, and $\beta = v/c$. In the limit $E \gg mc^2$, we also have $E \simeq pc$ (which is exactly true for a photon, of course; as the particle gets more relativistic, its rest mass becomes less and less important). Note that the Lorentz factor γ is often used to represent the energy; the mc^2 factor is implicitly carried along if you need "real" units.

Power-law distribution functions (motivated by cosmic rays) are often used:

$$f(E) = f_o E^{-s} \ , \quad E_1 \le E \le E_2 \tag{8.1}$$

or

$$n(\gamma) = n_o \gamma^{-s} \ , \quad \gamma_1 \le \gamma \le \gamma_2 \tag{8.2}$$

The exponent s depends on the system; the scaling constant f_o or n_o connects to the total number (or number density) of particles. That is, the total number of particles will be $N = \int f(E)dE = \int n(\gamma)d\gamma$. It follows, then, that $f(E)$ and $n(\gamma)$ have different units – watch out, this difference can bite you.[1]

[1] To be specific: the two DF's are equivalent if

$$f(E)dE = n(\gamma)d\gamma \ ; \quad f(E) = n(\gamma)d\gamma/dE$$

(so that we have the same number of particles "at $E = \gamma mc^2$"). For the specific power law case, above, this gives

$$f_o E^{-s} dE = n_o \gamma^{-s} d\gamma \ ; \quad f_o = n_o (mc^2)^{s-1}$$

8.2 Quick overview of the observations

We have seen that astrophysical plasmas contain two species. One species is the thermal interstellar gas which we have been considering. In this context, its important properties are that it seems to be well described by a thermal equilibrium distribution function (a Maxwell Boltzmann velocity distribution), and that the energy per particle is subrelativistic. (Recall temperatures range from $O(10)$K to $O(10^6)$K).

In addition, many astrophysical plasmas – including the galactic ISM – contain a significant population of highly relativistic particles which are *not* in a thermal distribution. We saw last term that we have direct and indirect evidence of these particles; I'll review the arguments here.

Baryons. Here we have direct evidence – these are the cosmic rays. They are mostly protons, but there is a heavy element component, with approximately solar abundances (so they come from processed material). They are very isotropic in arrival direction, probably at all energies (although arrival directions for the very highest energy particles remain uncertain).

• The baryon energy distribution is a power law, $N(E) \propto E^{-s}$, with a break at $E \sim 10^{15}$eV (the "knee"), and another at $E \sim 10^{19}$eV (the "ankle"). The exponent $s \sim 2.7$ below the ankle, and higher above. Comparison of the gyroradius to the scale of the galaxy suggests that the highest energy CR, above the ankle, are extragalactic, while the lower energy ones are galactic in origin.

Leptons. Here we have some direct evidence – the lepton component in the cosmic ray spectrum can be separated from the baryon component. The cosmic ray lepton distribution falls much more steeply than the baryon distribution, above energies ~ 1 GeV, so that its total contribution to the CR energy density is only $\sim 1\%$ that of the baryons.

In addition, there is also abundant *indirect* evidence fore highly relativistic electrons[2] throughout the universe. Synchrotron radiation – which we will study in detail in the next chapter – is common in many different settings. Because synchrotron radiation comes from highly relativistic particles (with $\gamma = E/mc^2 \gg 1$), we know immediately that synchrotron sources have a relativistic lepton component. Examples of this

[2] Well, really leptons; we'll discuss later whether we can distinguish electrons from positrons by their radiation signatures.

are the galactic disk; supernova remnants, both standard and "filled"; radio jets from compact stars in X-ray binaries; X- and γ-ray emission from pulsars; radio jets from active nuclei, and the radio lobes they create; diffuse synchrotron emission from the plasma in clusters of galaxies (including a few synchrotron-bright shocks created when two clusters collide); and quasars of course.

Looking ahead to the next chapter, we can note some important characteristics of synchrotron radiation. It requires magnetic fields and highly relativistic electrons. The spectrum from a single particle, with energy $E = \gamma mc^2$, peaks at a photon frequency

$$\nu_{sy} \sim \frac{3}{4\pi}\gamma^2 \frac{eB}{mc} \tag{8.3}$$

Thus, for a uniform B field, one particle energy γ maps directly to one (observed) photon frequency, ν. This single, radiating particle has an energy loss rate

$$\frac{dE}{dt} \simeq \frac{4}{3} c\sigma_T \gamma^2 \frac{B^2}{8\pi} \tag{8.4}$$

where $\sigma_T = 6.65 \times 10^{-25}\text{cm}^2$ is the Thompson cross section. Synchrotron radiation commonly has a power law spectrum, which tells us (assuming the B field is simple) that the underlying electron distribution is also power law (just as it is in galactic cosmic rays). Typical values for the photon spectrum are $j(\nu) \propto \nu^{-\alpha}$, with $\alpha \sim 0.5 - 1.0$; the associated electron distribution is $n(\gamma) \propto \gamma^{-s}$, with $s \sim 2.0 - 3.0$.

From(8.4), we see that higher energy electrons lose energy faster that the lower energy ones, because $dE/dt \propto E^2$; from this one can show that an initial electron power law spectrum will develop a break at the energy where the particle's radiative lifetime equals the age of the plasma. As the plasma gets older, this break will move to lower energies (and thus to lower photon frequencies).

8.3 Cosmic rays in the galactic setting

One of the big questions is, how are cosmic rays accelerated to such high energies, and how are they maintained there (why don't they eventually thermalize with the ISM)? Before we go there, let's recap a few points about cosmic rays from last fall (P425 notes).

Sources are thought to be two-fold. *Supernova remnants* have long been thought to be the main source of CR; I personally suspect that *pulsars* are probably also an important source. Still another possibility is that the highest energy CR may have an *extragalactic origin*.

Propagation and Trapping. Once generated, CR do not just fly freely through space. Becuase they are charged, they are connected to the ISM by their gyromotion, and by scattering on turbulent Alfven waves in the ISM. Thus the CR distribution we observe at earth may well have been seriously changed, relative to their "birth" distribution, by propagation and scattering through the ISM on their way to us. From nuclear abundances we learn that galactic CR typically spend most of their life in the galaxy, *not* in the disk, but rather in the more extended halo; and that its lifetime to escape from that halo ~ 20 Myr.

Losses. The leptons, being of smaller mass, are susceptible to radiative losses (synchrotron in the galactic magnetic field, inverse Compton scattering on whatever radiation is around) as well as Coulomb losses (scattering on the plasma component of the ISM). This also modifies the electron energy distribution, compared to the source, and of course reduces the net energy in the electron component of the CR.

8.4 Particle acceleration, overview

Next, we consider the question of particle acceleration: what is the origin of cosmic rays (which we observe directly at earth), and of the relativistic electrons in such sources as supernova remnants and radio jets (which we observe indirectly *via* their synchrotron radiation)? How are these charged particles accelerated to relativistic energies, and how are they maintained in a non-Maxwellian distribution? The short answer is, this must be done by electric fields, $\mathcal{E} \neq 0$. Magnetic fields do no work on the particles, and gravity is a conservative force (so that any energy a particle gains by going into a gravitational potential well must be lost again when it leaves the well). However, is it not easy to maintain large-scale electric fields in space, where the abundant free charges in astrophysical plasmas will want to short out any static field. Two types of particle acceleration models exist — which I tend to call "first stage" and "second stage" mechanisms.

8.5 Particle acceleration, first stage mechanisms

One possibility is that an ordered, large-scale $\mathcal{E}$ field is maintained in the region of some massive object, like a star (thinking of solar flares) or a pulsar or an accretion disk, where dynamic processes can maintain a strong dynamo; these are called first stage mechanisms. In this situation there is no minimum energy threshold for particle acceleration; any charged particle dropped in a region with $\mathcal{E} \neq 0$ will be energized. There may, however, be an upper limit to the energy that can be reached – due to the physical size of the system (and the consequent limit on the overall potential drop, $\int \mathcal{E} \cdot d\mathbf{s}$). There are a couple basic types of models here: reconnection sites and unipolar dynamos.

8.5.1 Magnetic reconnection

You saw this last term; I'll just store a recap here. Consider a region in a plasma in which the direction of the magnetic field reverses over a small spatial scale – for instance, the neutral sheet in the earth's magnetotail, or the base of a flux tube footed in the solar photosphere. The magnetic field in this region will find a lower-energy state by "reconnecting", that is by changing the magnetic field configuration. Such a region can be the site of rapid conversion of stored magnetic energy to kinetic and internal energy of the plasma, Reconnection is believed to be important in solar flares, and has been observed in the earth's magnetotail.

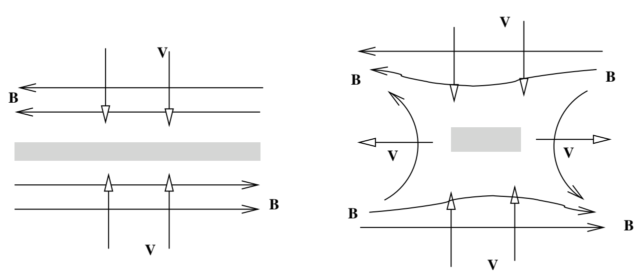

Figure 8.1 The geometry of magnetic reconnection. Left, a simple system before reconnection; the field lines have opposing directions and meet in a central current sheet (called a neutral sheet), shaded in this figure. Right, with reconnection ongoing; some field lines have changed their topology, and plasma is driven out of the neutral sheet across the reconnected field lines. Think: which way to the current and **E** field point in the neutral sheet?

The reason reconnection is important for particle acceleration, is that the neutral sheet (where the magnetic field goes through zero) contains an ordered electric field, of strength

$$\mathcal{E} = -\frac{c}{4\pi\sigma}\nabla \times \mathbf{B} - \frac{1}{c}\mathbf{v} \times \mathbf{B} \tag{8.5}$$

(This is just Ohm's law, combined with Maxwell's equations; check the section earlier on flux freezing). Here, σ is the electrical conductivity (not a cross section). In the cartoons above, the $\mathcal{E}$ field will be in the plane of the neutral sheet and perpendicular to the paper. The maximum particle energy that can be reached, then, is set by the potential drop across the neutral sheet, $\Delta V = \int \mathcal{E} \cdot d\mathbf{l}$.

8.5.2 Unipolar dynamos

This is a jargon-word for massive, rotating, compact objects – pulsars and accretion disks – which can support strong fields. (Refer back to Fig 9.3 of your P425 notes.) If a magnetic field is tied to the rotating matter or plasma (say by a high conductivity, so that the field is approximately flux-frozen), the rotation will cause a field $\mathcal{E} = \frac{1}{c}\mathbf{v}_{rot} \times \mathbf{B}$ to be seen by a local observer.

The numbers that are in principle reached by rotating compact objects are impressive. For pulsars, flux-freezing considerations estimate $B \simeq 10^{12}$G if the field was originally the stellar field. For a dipole field, this would give $B(r) \simeq 10^{12}(R_{ns}/r)^3$G if R_{ns} is the neutron star radius. The consequent $\mathcal{E}$ field and potential drop are then

$$\mathcal{E} \simeq \frac{2 \times 10^{14}}{P}\left(\frac{R_{ns}}{r}\right) \mathrm{V/m}; \tag{8.6}$$

if P is the pulsar's period;

$$\Delta V = \int \mathcal{E} \cdot d\mathbf{l} \simeq 2 \times 10^{13} \frac{(R_{ns}/1\mathrm{km})}{(P/0.03\mathrm{sec})} \mathrm{V} \tag{8.7}$$

(This is scaled to the Crab pulsar; millisecond pulsars would reach $\sim 10^{15}$V). For an accretion disk around a black hole, conventional estimates replace R_{ns} by $r_g = GM_{bh}/c^2$, and note that the inner parts of an accretion disk rotate at nearly lightspeed. If the magnetic field in the disk $\sim 10^3$G or so, the potential drop can reach10^{19}V.

On the other hand, it is not obvious that these high fields can be maintained – in fact it's very unlikely. The picture we have presented of a rotating, magnetized object is a vacuum argument. But the atmospheres of such objects are likely to have some amount of free change, which may well short out a large part of these maximum predicted potential drops. For one example, think about a relativistic particle being accelerated along the magnetic field line in a pulsar by these strong $\mathcal{E}$ fields. As it gets accelerated, it radiates – and the γ-ray photons it radiates can pair produce in the pulsar's strong magnetic field. The newly created electron-positron pairs shield most of the rotation-induced $\mathcal{E}$ field ... leading to a net particle energy only $\sim 10^{-6}$ or so of the maximum predicted by (8.6).

These two families of models, reconnection and unipolar dynamos, represent most of the models of particle acceleration by large-scale electric fields. What are the important characteristics of these models, as far as their relevance to particle acceleration? First, all charges which see $\mathcal{E}$ are accelerated. Second, the maximum energy that can be reached is given by the potential drop; but real-world effects, mostly related to local plasmas (shorting out part of the potential drop, or providing turbulence which can stop the speeding particles before they reach $E \sim e\Delta V$). Third, due to the effects just listed, much of the available energy goes to heating the plasma rather than accelerating a small fraction of the charged particles to high energies. The final values of the output particle energies depend on these details, of course. Observations of solar and magnetospheric reconnection suggest that some fraction of the charged particles are, indeed, accelerated to well above thermal energies; but not to relativistic energies. (The output energy may well be larger in pulsars and galactic-nucleus accretion disks; but we can only work from inference there). Thus, these mechanisms are often thought of as a "first stage" in the acceleration process; stochastic methods may take these "seed" particles and boost them to much higher energies.

8.6 Particle acceleration, second stage mechanisms

The other class of acceleration mechanisms invokes stochastic electric fields, such as are found in plasma turbulence. If the particles couple efficiently to the turbulent $\mathcal{E}$ fields, they can gain energy from the turbulence. There are again a couple of versions of this: true turbulence (disordered plasma motions, for instance somewhere in the dynamic ISM), or shock acceleration (using shock physics to drive the turbulence).

The fundamental idea of stochastic particle acceration was invented by Fermi in 1949; his physical picture was rather naive, but described the basic stochastic mechanism quite clearly. More recent work has improved the physical description of the scattering mechanism, while retaining the basic idea.

We have already seen the most likely scattering mechanism: resonant interaction with Alfven waves. In this situation there is a minimum particle energy which can interact with the waves, as we saw last term. Therefore, these models can be thought of as second stage mechanisms, in that they take "seed" particles created by first-stage mechanisms and accelerate them to very high energies. The maximum energy which can be reached here is also finite, and again tied to the size of the system (which determines the maximum Alfven wavelength which can exist, and also determines the rate at which particles can leak out of the acceleration region). But these upper limits can be higher, in some astrophysical settings, than those provided by first-stage mechanisms.

8.6.1 Fermi acceleration

This is the original picture. Think back to last term, when we talked about "magnetic mirrors". That is; for a particle moving in a region of spatially changing magnetic field, the magnetic moment, $\mu = p_\perp^2/2\gamma m B$ is conserved, as long as the time during which the particle sees the field to change is long compared to the particle's gyroperiod. But since $p^2 = p_\perp^2 + p_\parallel^2$ is fixed in the absence of external forces, $p_\perp^2 \leq p^2$. Thus, there is a maximum value of B which is allowed; the particle is kept out of high-field regions. This effect is called magnetic mirroring.

Now, to apply this to cosmic rays, envision a field of randomly moving mirrors (we can think of them as interstellar clouds, or as clumps of high magnetic field) on which the relativistic particles or cosmic rays scatter elastically. We want the effect of many collisions with these mirrors, on a particle distribution. Let the mirrors have random velocity v_m, and average spacing L. If a particle is moving faster than the mirrors, it can undergo either overtaking or head-on collisions. In a single collision, the particle (mass m, velocity v) suffers velocity and kinetic energy changes,

$$\begin{aligned} &\mathrm{head-on:} \\ &\quad \Delta v \simeq 2v_m; \;\; \Delta E \simeq 2mv_m(v+v_m) \\ &\mathrm{overtaking:} \\ &\quad \Delta v \simeq -2v_m; \;\; \Delta E \simeq 2mv_m(v-v_m) \end{aligned} \tag{8.8}$$

But, the rate at which the particle suffers headon collisions is $\simeq (v+v_m)/L$; its rate of overtaking collisions is $\simeq (v-v_m)/L$. Thus, the net rate of change of the particle's energy is

$$\begin{aligned} \frac{dE}{dt} \simeq & \frac{(v+v_m)}{L} 2mv_m(v+v_m) \\ & - \frac{(v-v_m)}{L} 2mv_m(v-v_m) \end{aligned} \tag{8.9}$$

and this collects to the final form,

$$\frac{dE}{dt} \simeq 16\frac{E}{t_{coll}}\left(\frac{v_m}{v}\right)^2 \tag{8.10}$$

where we have written $t_{coll} = L/v$. While I wouldn't take the factor 16 too seriously, the fundamental form is: $dE/dt \propto E/t_{coll}$, which is the standard result for Fermi acceleration.

A couple of features are worth noting. One, this is a test particle approach; the energy loss of the mirrors is not taken into account. Two, the fractional energy gain per collision is small, since $v_m \ll v \simeq c$; thus, the acceleration time $t_{acc} \sim E/(dE/dt) \sim (v/v_m)^2 t_{coll}$.

What particle spectrum is predicted by this simple model? One way to find this is as follows. From the kinematics above, we found that the average energy gain per collision is $\Delta E \sim E(v_m/c)^2$ if the particles are relativistic. Thus, after p collisions, a particle which started at E_o will have energy E_p after p bounces:

$$E_p \simeq E_o\left(1+\frac{v_m^2}{c^2}\right)^p \tag{8.11}$$

which can be written,

$$\ln\frac{E_p}{E_o} = p\ln\left(1+\frac{v_m^2}{c^2}\right) \simeq p\frac{v_m^2}{c^2}. \tag{8.12}$$

Now, let the particles have some chance, η, of escaping from the acceleration region (something must end the acceleration process, after all!). If the escape time $\sim \tau$, then $\eta \sim t_{coll}/\tau$. But, the number of particles at E_p, $E_p f(E_p)$, is just the starting number, N_o, times the probability of the particle staying in the system for p bounces:

$$E_p f(E_p) = N_o P(p) = N_o(1-\eta)^p \tag{8.13}$$

which we can write as

$$\begin{aligned}\ln[E_p f(E_p)] &= \text{constant} + p\ln(1-\eta)\\ &\simeq \text{const} - p\eta\end{aligned} \tag{8.14}$$

Combining this with (8.12), eliminating p and dropping the p subscript, we find the predicted spectrum:

$$f(E) \propto E^{-(1+\eta c^2/v_m^2)} \tag{8.15}$$

Thus, this model – Fermi acceleration plus a constant probability of escape from the system – results in a power law spectrum, as is observed in cosmic rays and elsewhere.. However, as this model stands, the exponent of the power paw, $s = 1 + \eta c^2/v_m^2$, is a sensitive function of the mirror parameters and the escape time. This is not very appealing, since the observed values of s fall in the range $2 \lesssim s \lesssim 3$ almost everywhere (cosmic rays, supernova remnants, radio galaxies, etc.).

8.6.2 Plasma turbulence: Alfven waves

Modern work has replaced Fermi's picture of moving magnetic mirrors with small-scale structures which can scatter particles: most likely these are Alfven waves.

I'm sure you remember these waves, from last term. They are transverse waves, which are not compressive, and which propagate (in the simplest case) along the magnetic field. Thus, they can be thought of as propagating wiggles in the field lines. Their dispersion relation is $\omega = kv_A$, with $v_A = B/\sqrt{4\pi\rho}$. Charged particles interact resonantly with Alfven waves. A particle moving along $\mathbf{B}$ at some velocity v sees a Doppler shifted frequency $\omega' = \omega(1 - v/v_A) = \omega - kv$. Now, the particle will interact with the fluctuating E field of the wave; if the particle "stays in phase" with this fluctuating wave, it will undergo a strong interaction. But this happens if the Doppler shifted wave frequency is close to the particle's natural frequency, its gyrofrequency. That is, the interaction is strong when

$$\omega - kv = \pm\Omega(\gamma) \tag{8.16}$$

For relativistic particles, with $v \gg v_A$, this condition is equivalent to an equality between the particles gyroradius and the wave's wavelength:

$$r_L(\gamma) = \gamma\frac{mc^2}{eB} \simeq \lambda_{res}(\gamma) \tag{8.17}$$

Numerically, note that particles with $\gamma \sim 10^3$ in a field $B \sim 1\ \mu$G have a gyroradius – and thus a resonant Alfven wavelength – on the order of an AU.

If the Alfven waves are turbulent (say, just arising from a turbulent background plasma, such as the ISM), the picture above carries over directly; v_m becomes v_A, and the concept of a scattering length L must be replaced with the wave energy density and some measure of the scattering cross section, similar to that used above. This process is slow in many applications; it has $dE/dt \propto v_A^2/c^2$ (and thus is classically a "second order Fermi process"). In addition, because it relies on the wave-particle resonance, a particle at energy γ only sees waves at $\lambda_{res}(\gamma)$ – which can be a small fraction of the total energy in the turbulence.

8.6.3 Turbulent shock acceleration

A faster version of turbulent acceleration ties the turbulence to shock fronts. A shock must have $v_2 < v_1$ (the post-shock velocity must be less that the pre-shock velocity; right?). Thus, in the frame of the shock, the flows on each side are converging. If the pre-shock and post-shock plasma also carry Alfven turbulence, a particle trapped in the shock region and bouncing back and forth will undergo Fermi acceleration, but will suffer only head-on collisions. With this picture, we can specify the parameters in the above Fermi analysis. We need two facts:

• The escape probability is the ratio of downstream to upstream fluxes: $\eta \simeq 4n_{rel}v_2/n_{rel}v = 4v_2/c$.

• The energy gain of a particle per scattering turns out to be

$$E' = E\frac{[1 + v_{e1}(v_1 - v_2)\cos\theta_{e1}/c^2]}{[1 + v_{e2}(v_1 - v_2)\cos\theta_{e2}/c^2]} \quad (8.18)$$

if v_{e1}, v_{e2}, θ_{e1} and θ_{e2} are the velocities and angles of the particle (electron, say), before and after it is scattered. After p bounces, we have

$$\ln\frac{E_p}{E_o} \simeq \frac{4}{3}p\frac{(v_1 - v_2)}{c} \quad (8.19)$$

With these, we can collect the algebra as above and find the spectrum predicted by this model:

$$f(E) \propto E^{-s}; \quad s = \frac{v_1 + 2v_2}{v_1 - v_2} \quad (8.20)$$

Now, for a strong shock, $v_1 = 4v_2$, which predicts $s = 2$; higher s values will be produced if the velocity jump in the shock is < 4.

8.6.4 Energy limits for Alfven acceleration

What are the limits on particle energies that can be scattered and accelerated by Alfven waves? The particle-wave resonant condition (8.17) makes this easy to answer. For the lower limit, we noted last term that Alfven waves can only exist for frequencies $\omega < \Omega_p = eB/m_pc$, the proton gyrofrequency. This translates to a lower limit on the particles energies that can "see" the waves (given in chapter 8 of P425 notes).

The highest particle energy which can possibly be accelerated by Alfven waves is determined by the maximum wavelength that can exist in the system (which can't be any larger, clearly, than the physical size of the system). In practice, however, particle acceleration is limited by losses occuring at the same time as the acceleration. If the losses are due to synchrotron radiation, higher energy particles lose their energy more rapidly (equation 8.4); this leads to a second upper limit on the energy range of accelerated particles.

Key points

• Cosmic rays: in the galactic setting

• Relativistic electrons: indirectly "seen"

• First stage particle acceleration: where, what energies?

• Fermi acceleration, "classical"

• Alfven wave acceleration, shock and/or turbulent